The ‘half moon’ singularity

Discussion at annual TORUS Collaboration meeting

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Contents

- Introduction of Faddeev equations.
- What is the ‘half moon’ singularity?
- Traditional ways to deal with this singularity.
- New outlook on the ‘half moon’ problem.

Faddeev equations for Three Identical Bosons

Nd breakup

$$T|\phi\rangle = tP|\phi\rangle + tPG_0T|\phi\rangle.$$

Amplitude is[a]

$$\langle\vec{p}\vec{q}|U_0|\phi\rangle = \langle\vec{p}\vec{q}|(1+P)T|\phi\rangle.$$

[a] $|\phi\rangle = |\varphi_d\vec{q}_0\rangle$.

Expression with explicit singularity (3D)

$$G_0 = \frac{1}{E + i\epsilon - \frac{1}{m}\left(p''^2 + \frac{3}{4} q''^2\right)},$$

$$\langle \vec{p'} \vec{q} | P | \vec{p''} \vec{q''} \rangle = \delta(\vec{p'} + \vec{\pi}_1)\, \delta(\vec{p''} - \vec{\pi}_2) + \delta(\vec{p'} - \vec{\pi}_1)\, \delta(\vec{p''} + \vec{\pi}_2),$$

$$\langle \vec{p} \vec{q} | tP | \varphi_d \vec{q_0} \rangle = T_0(\vec{p}, \vec{q}; \vec{q_0}), \quad \vec{\pi}_1 = \frac{1}{2}\vec{q} + \vec{q''}, \quad \vec{\pi}_2 = \vec{q} + \frac{1}{2}\vec{q''}.$$

$$t_s(\vec{p}, \vec{\pi}_1; z) = t(\vec{p}, \vec{\pi}_1; z) + t(\vec{p}, -\vec{\pi}_1; z).$$

$$T(\vec{p}, \vec{q}; \vec{q_0}) = T_0(\vec{p}, \vec{q}; \vec{q_0}) + \int d^3 q'' \frac{t_s\left(\vec{p}, -\vec{\pi}_1; E - \frac{3}{4m} q^2\right) T(\vec{\pi}_2, \vec{q''}; \vec{q_0})}{E + i\epsilon - \frac{1}{m}\left(q^2 + q''^2 + \vec{q} \cdot \vec{q''}\right)}.$$

After partial wave decomposition

$$\cdots \int dq''\, q''^2 \int_{-1}^{1} dx \frac{\cdots}{E + i\epsilon - \frac{1}{m}\left(q^2 + q''^2 + qq''x\right)}.$$

Integration over x

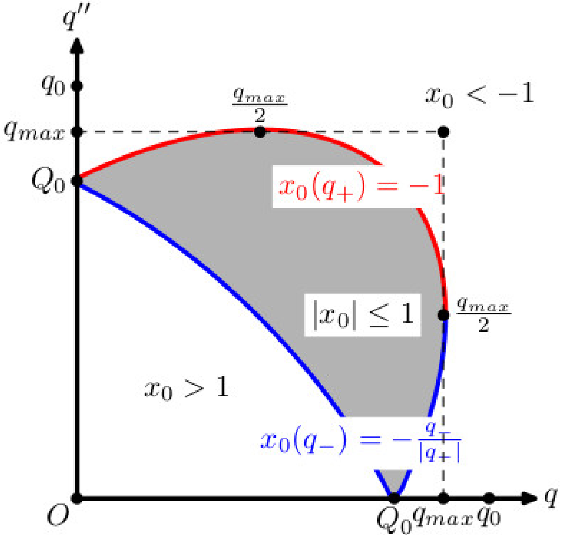
$\forall q < q_{max}, \quad \exists\{q'', x_0\} : \quad q^2 + q''^2 + qq''x_0 = mE, \Rightarrow$ singularity.

$$\cdots \Rightarrow \ln\left|\frac{1 + x_0}{1 - x_0}\right|.$$

Sometimes this singularity is located just at the end of the integration region ($x_0 = \pm 1$).

$$q_{max} = \sqrt{\frac{4m}{3}E}, \qquad x_0 = \frac{mE - q^2 - q''^2}{qq''}.$$

Illustration



Traditional ways to deal with this singularity

- Switch to the complex plain of q'' (Hetherington & Schick). No way! We don't have an analytic continuation of our finctions to the complex plain.
- Subtraction (W. Glöckle & H. Witała).
- Using splines (W. Glöckle, Ch. Elster, & H. Liu):

$$f(y) \Rightarrow a_0 + a_1\, y + a_2\, y^2 + a_3\, y^3 \Rightarrow \text{semi-analytical integration.}$$

W. Glöckle's suggestion

References

Initial: H. Witała and W. Glöckle. // Eur. Phys. J. A **37**, 87-95 (2008).

In 3D: Ch. Elster, W. Glöckle, and H. Witała. // Few-Body Syst. (2009) 45: 1–10. DOI: 10.1007/s00601-008-0003-6.

The idea

Detangle q and q'' in the denominator using the internal δ-functions.

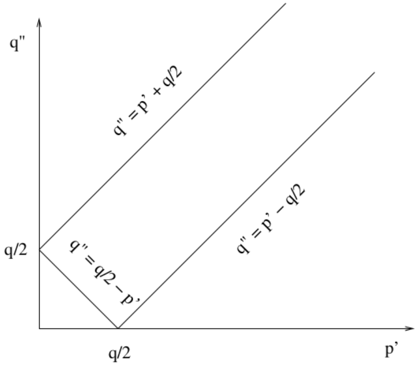
Example:

$$\delta(p' - \pi_1) = \frac{2p'}{qq''}\delta(x - X)\,\Theta(1 - |X|), \qquad X = \frac{{p'}^2 - \frac{1}{4}q^2 - {q''}^2}{qq''}.$$

W. Glöckle's suggestion (cont.)

The Results:

$$\cdots \int dq''\, q''^2 \frac{\cdots\ \Theta(\cdots)\cdots}{E + i\epsilon - \frac{3}{4m}q''^2 - E_d} + \cdots \int dp'\, p'^2 \frac{\cdots\ \Theta(\cdots)\cdots}{E + i\epsilon - \frac{1}{m}(p'^2 + \frac{3}{4}q^2)}.$$



Due to Θ-functions, integration only outside of the rectangular region. No coupled singularities!