Methods for Vertex Integrals of Coulomb Potentials

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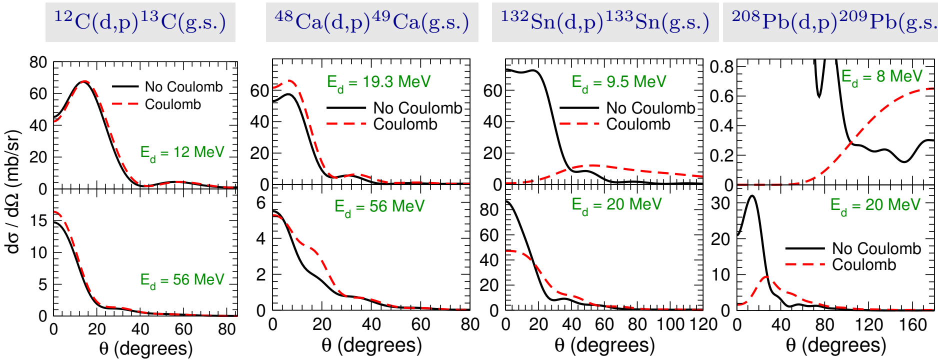
Significance of Coulomb



Coulomb dominates!

Treatment of Coulomb: Coordinate space

Coulomb potential: $$V^C(r) = \frac{Z_1 Z_2 e^2}{r}$$

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Screened Coulomb potential:

$$\lim_{R\to\infty} V^R(r) = \lim_{R\to\infty} V^C(r)\,\xi(r,R)$$

where $\xi(r,R)$ is a damping function
R is screening radius

Treatment of Coulomb: Momentum space (Conventional)

✍ A. Deltuva *et al.*, PR**C71**, 054005 (2005).

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with V^N: Nuclear Potential
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$$T_l^{(R)} \longrightarrow f_l^{(R)}$$

Re-normalized Coulomb scattering amplitude:

$$f_l^{(C)} \longrightarrow \lim_{R\to\infty} e^{i\phi(k,R)}\ f_l^{(R)}\ e^{i\phi(k,R)} \equiv \lim_{R\to\infty} \tilde{f}^{(R)}(k)$$

where $\phi(k, R)$ is the renormalization phase factor.

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Problem: As R $\to \infty$, $\phi(k,R) \to \infty \implies \tilde{f}^{(R)}(k)$ will not converge!!

New method to handle coulomb in momentum space

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✓ Separable potential: $V_l^n(q',q) = \sum_{i,j=1}^{n} h_{l,i}(q')\,\lambda_{l,ij}\,h_{l,j}(q)$

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$$Z_l^{SC}(p, p_\alpha) = \int \frac{dp'\, {p'}^2}{2\pi^2} V_l(p, p')\; \psi^C_{p_\alpha l}(p')$$

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$\psi^C_{p_\alpha l}(p')$: coulomb wave function

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Coulomb wave function $[\psi^C_{p_\alpha l}(p')]$

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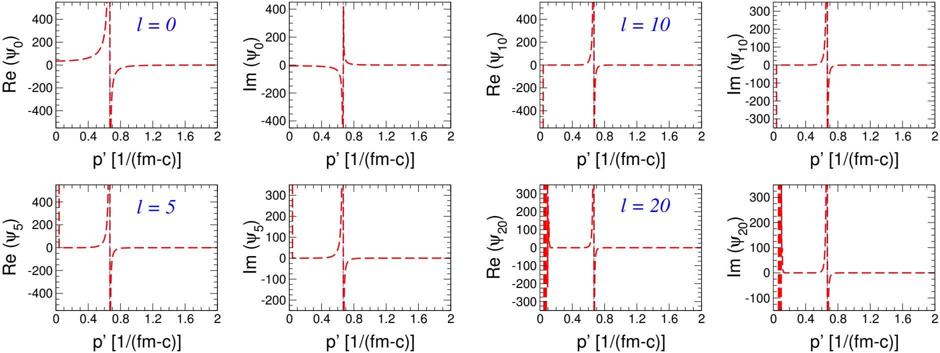
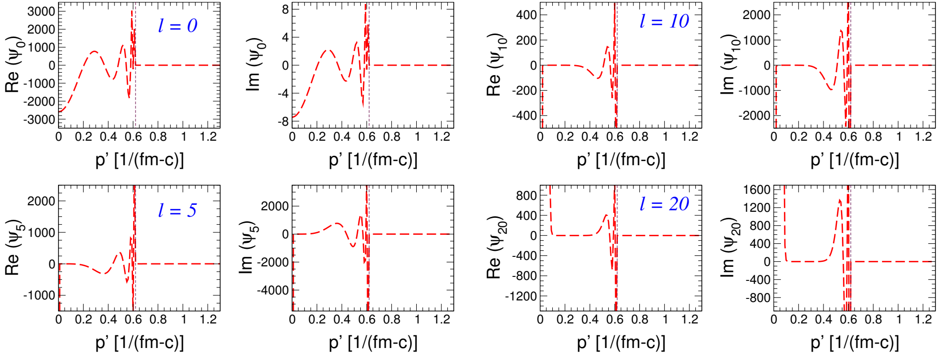
$$\psi^C_{p_\alpha l}(p') = \frac{-4\pi\, e^{-\eta_\alpha \pi/2}}{p'} \left(\frac{(p'+p_\alpha)^2+\gamma^2}{4p'p_\alpha}\right)^l \times \Gamma(1+i\eta_\alpha)\, e^{i\alpha_l}$$
$$\times \lim_{\gamma\to+0} \mathrm{Im}\left\{\left[e^{-i\alpha_l}\, \frac{(p'+p_\alpha+i\gamma)^{i\eta_\alpha-1}}{(p'-p_\alpha+ii\gamma)^{i\eta_\alpha+1}}\right.\right.$$
$$\left.\left.\times\, {}_2F_1\left(-l, -l-i\eta_\alpha; 1-i\eta_\alpha; \frac{(p'-p_\alpha)^2+\gamma^2}{(p'+p_\alpha)^2+\gamma^2}\right)\right] + \gamma\left[\ldots\ldots\right]\right\}$$

Coulomb wave function $[\psi^C_{p_{\alpha l}}(p')]$: Functional Behaviour

$p+{}^{12}C$ @ $E_{cm} = 10$ MeV:



$\mathrm{p}+{}^{208}\mathrm{Pb}$ @ $\mathrm{E_{cm}} = 8$ MeV:



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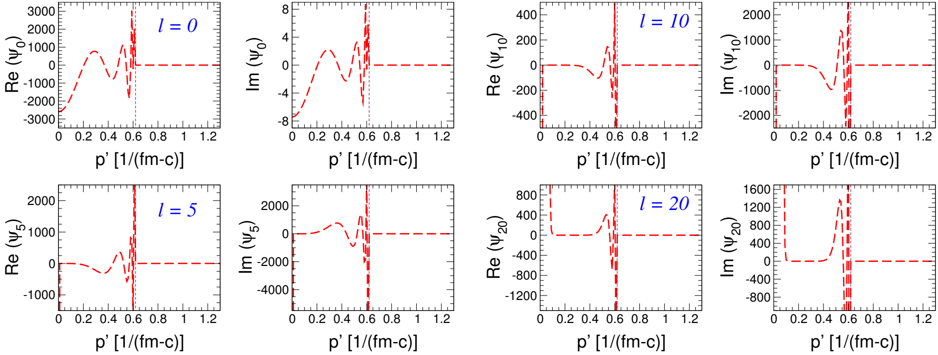

Artificial singularity at low momenta is because of wrong expansion of hypergeometric function!!

Coulomb wave function $[\psi^{C}_{p_\alpha l}(p')]$: 2 Definitions

$$\psi^{C}_{p_\alpha l}(p') = \frac{-4\pi\, e^{-\eta_\alpha \pi/2}}{p'} \left(\frac{(p'+p_\alpha)^2+\gamma^2}{4p' p_\alpha}\right)^{l} \times \Gamma(1+i\eta_\alpha)\, e^{i\alpha_l}$$

$$\times \lim_{\gamma\to+0} \mathrm{Im}\left\{ \left[e^{-i\alpha_l} \frac{(p'+p_\alpha+i\gamma)^{i\eta_\alpha-1}}{(p'-p_\alpha+ii\gamma)^{i\eta_\alpha+1}} \right.\right.$$

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$\psi^{C}_{p_\alpha l}(p')$ at low & high momenta:

$$\psi^{C}_{p_\alpha l}(p') = -2\pi\, e^{-\eta_\alpha \pi/2} (p'\, p_\alpha)^{l} \left[\frac{\Gamma(l+1+i\eta_\alpha)\Gamma(\frac{1}{2})}{\Gamma(l+\frac{3}{2})}\right]$$

$$\times \lim_{\gamma\to+0} \left\{ \left[\left(\frac{2(p'^2-(p_\alpha+i\gamma)^2)^{i\eta_\alpha}}{(p'^2+p_\alpha^2+\gamma^2)^{l+i\eta_\alpha+1}}\right) \left(\frac{\eta_\alpha(p_\alpha+i\gamma)}{p'^2-(p_\alpha+i\gamma)^2} - \frac{\gamma(l+i\eta_\alpha+1)}{p'^2+p_\alpha^2+\gamma^2}\right) \right.\right.$$

$$\left.\left. \times\, {}_2F_1\left(\frac{l+i\eta_\alpha+2}{2}, \frac{l+i\eta_\alpha+1}{2};\, l+\frac{3}{2};\, \frac{4p'^2 p_\alpha^2}{(p'^2+p_\alpha^2+\gamma^2)^2}\right) \right] + \gamma\left[\,\ldots\ldots\,\right] \right\}$$

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Switch: $\dfrac{4{p'}^2 p_\alpha^2}{({p'}^2+p_\alpha^2+\gamma^2)^2} = \dfrac{(p'-p_\alpha)^2+\gamma^2}{(p'+p_\alpha)^2+\gamma^2}$

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Approach to test new method

$$Z_l^{SC}(p, p_\alpha) = \int \frac{dp'\, p'^2}{2\pi^2} V_l(p, p')\ \psi^C_{p_\alpha l}(p')$$

✓ $\psi^C_{p_\alpha l}(p')$: Study functional behaviour

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$$V(\mathbf{p}, \mathbf{p}') = -\frac{2\mu_{12}\, Z_1\, Z_2\, e^2}{(\mathbf{p} - \mathbf{p}')^2 + \mathfrak{K}^2} \longrightarrow \text{Yamaguchi form}$$

$$V_l(p, p') = -\frac{\eta_\alpha\, p_\alpha\, Q_l(\xi)}{pp'} \quad \left(\text{where, } \xi = \frac{p^2 + p'^2 + \mathfrak{K}^2}{2pp'}\right)$$

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- ✓ $p = p_\alpha$

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Integrand: Functional Behaviour

$V_l(p_\alpha, p') \longrightarrow$ Yamaguchi form

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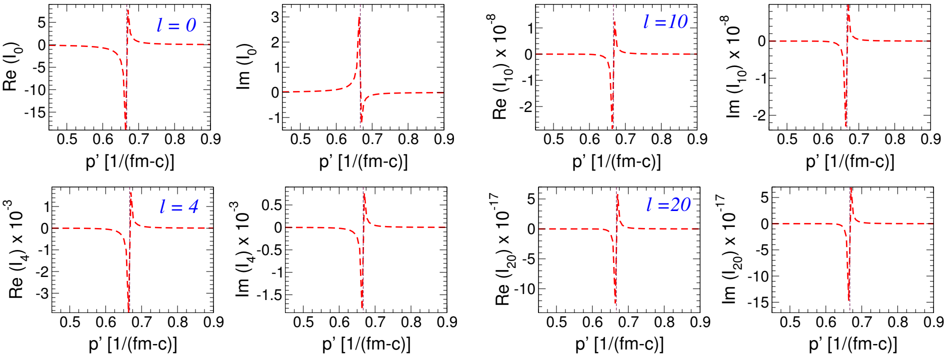
p+^{12}C @ E_{cm} = 10 MeV:



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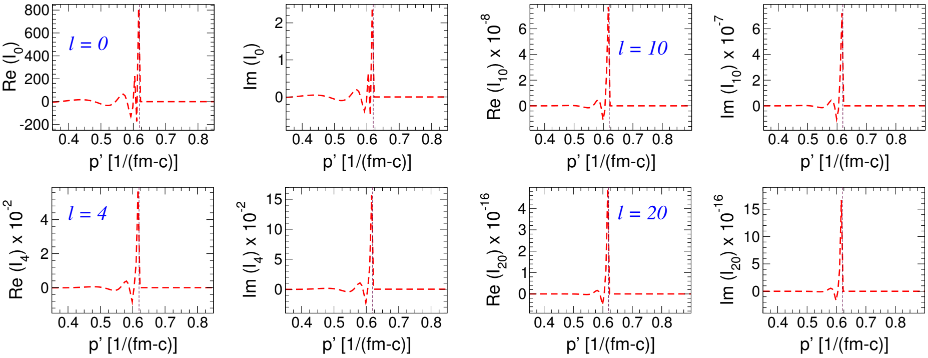

Behaviour of Integral

Integral:

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$$\int_0^\infty \frac{dp'\, p'^2}{2\pi^2}\, V_l(p_\alpha, p')\, \psi^C_{p_\alpha l}(p') = \int_0^{p_\alpha - \Delta} \ldots.. + \int_{p_\alpha - \Delta}^{p_\alpha + \Delta} \ldots.. + \int_{p_\alpha + \Delta}^{\infty} \ldots..$$

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Regular (first and last integrals)

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Irregular (containing singularity)

$$\int_0^\infty \frac{dp'\,p'^2}{2\pi^2}\,V_l(p_\alpha,p')\,\psi^C_{p_\alpha l}(p') = \int_0^{p_\alpha-\Delta} \ldots\ldots + \boxed{\int_{p_\alpha-\Delta}^{p_\alpha+\Delta} \ldots\ldots} + \int_{p_\alpha+\Delta}^{\infty} \ldots\ldots$$

Regular

Regularization: Gel'Fand-Shilov Method

$$\int_{p_\alpha - \Delta}^{p_\alpha + \Delta} \ldots\ldots \quad \longrightarrow \quad \int_{p_\alpha - \Delta}^{p_\alpha + \Delta} \frac{\phi(p' - p_\alpha)}{(p' - p_\alpha + i\gamma)^{i\eta_\alpha + 1}} \, dp'$$

Regularization: Gel'Fand-Shilov Method

$$\int_{p_\alpha-\Delta}^{p_\alpha+\Delta} \ldots\ldots \quad \longrightarrow \quad \int_{p_\alpha-\Delta}^{p_\alpha+\Delta} \frac{\phi(p'-p_\alpha)}{(p'-p_\alpha+i\gamma)^{i\eta_\alpha+1}}\, dp'$$

$$= \int_{p_\alpha-\Delta}^{p_\alpha+\Delta} dp' \, \frac{\left[\phi(p'-p_\alpha) - \phi(p_\alpha) \; - \; (p'-p_\alpha+i\gamma)\,\phi'(p_\alpha)\right]}{(p'-p_\alpha+i\gamma)^{i\eta_\alpha+1}}$$

$$+ \frac{i\phi(p_\alpha)}{\eta_\alpha}\left[(\Delta+i\gamma)^{-i\eta_\alpha} - (-\Delta+i\gamma)^{-i\eta_\alpha}\right]$$

$$+ \frac{\phi'(p_\alpha)}{(1-i\eta_\alpha)}\left[(\Delta+i\gamma)^{1-i\eta_\alpha} - (-\Delta+i\gamma)^{1-i\eta_\alpha}\right]$$

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$V_l(p_\alpha, p') \longrightarrow$ Yamaguchi form

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$$
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Mathematica Results

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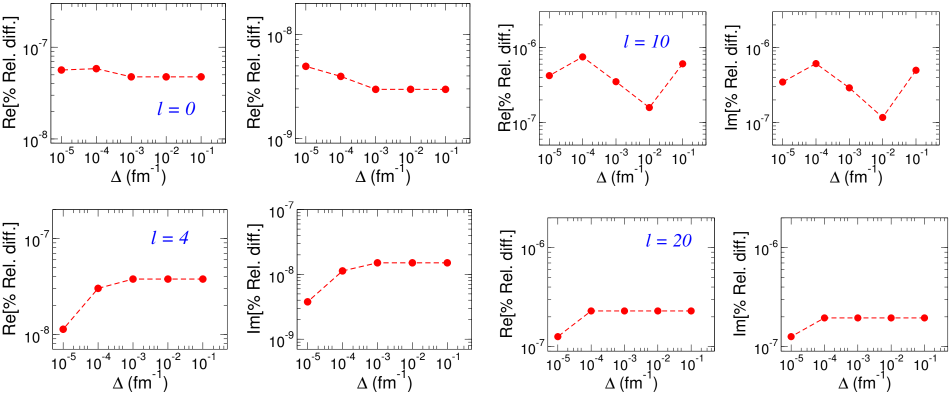

TORUS COLLABORATION

Dependence of $Z_l^{SC}(p_\alpha, p_\alpha)$ on Δ

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p+^{208}Pb @ E_{cm} = 8 MeV:

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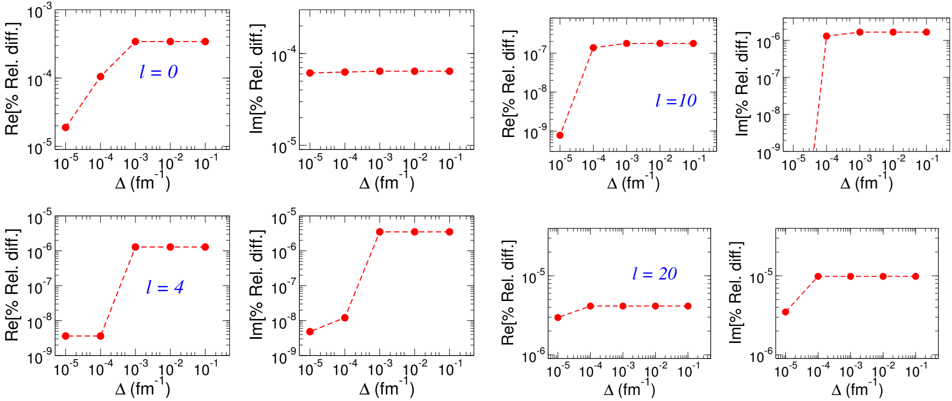

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- $Z_l^{SC}(p, p_\alpha)$

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$Z_l^{SC}(p, p_\alpha)$: Mathematica Results

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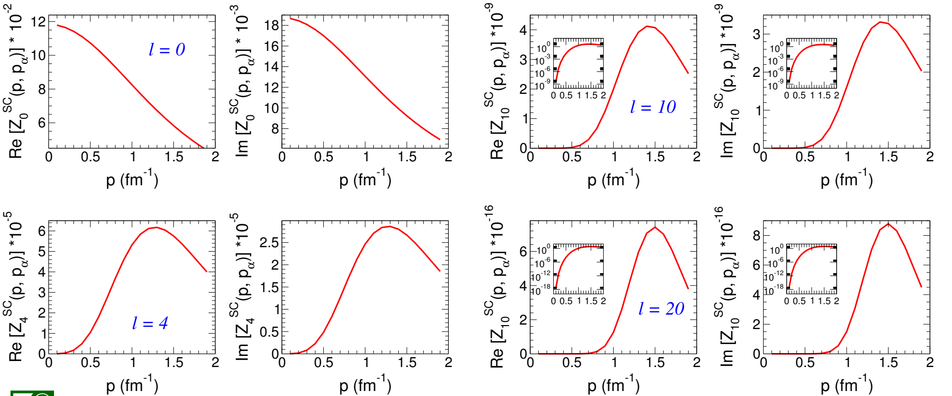
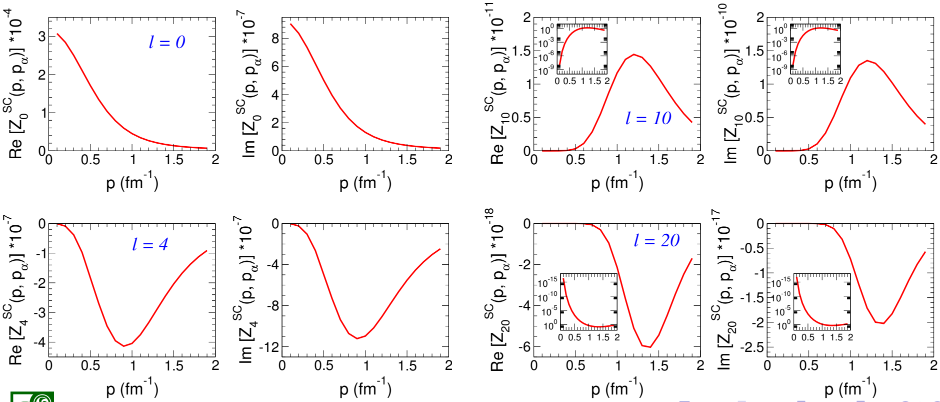


$V_l(p, p') \longrightarrow$ Yamaguchi form

$$Z_l^{SC}(p, p_\alpha) = \int_0^\infty \frac{dp' \, p'^2}{2\pi^2} V_l(p, p') \, \psi^C_{p_\alpha l}(p')$$

p+^{208}Pb @ E_{cm} = 8 MeV:



Approach to test new method

$$Z_l^{SC}(p, p_\alpha) = \int \frac{dp'\, {p'}^2}{2\pi^2} V_l(p, p')\ \psi^C_{p_\alpha l}(p')$$

- ✓ $\psi^C_{p_\alpha l}(p')$: Study functional behaviour
- ✓ $V_l(p, p')$:

Study in progress!

1. **Realistic Case**: Separable potential (Provided by L. Hlophe & Prof. Elster, OU).
2. **Demonstration**:

$$V(\mathbf{p}, \mathbf{p}') = -\frac{2\mu_{12}\, Z_1\, Z_2\, e^2}{(\mathbf{p} - \mathbf{p}')^2 + \mathfrak{K}^2} \longrightarrow \text{Yamaguchi form}$$

$$V_l(p, p') = -\frac{\eta_\alpha\, p_\alpha\, Q_l(\xi)}{pp'} \quad \left(\text{where, } \xi = \frac{p^2 + {p'}^2 + \mathfrak{K}^2}{2pp'}\right)$$

- ✓ $p = p_\alpha$

$$Z_l^{SC}(p_\alpha, p_\alpha) = \int \frac{dp'\, {p'}^2}{2\pi^2} V_l(p_\alpha, p')\ \psi^C_{p_\alpha l}(p')$$