Coulomb distorted nuclear matrix elements in momentum space.
II. Computational aspects

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(d, p) reactions

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Effective Three-Body problem

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Faddeev equations with Coulomb interaction and nucleus excitation

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Optical Short Range (Nuclear) Potentials in Separable Form preferred

- Faddeev equations ⇒ preferrably solved in momentum space.
- (d, p) reaction with nucleus excitation ⇒ Separable Optical Short Range Potential.
- Coulomb interaction ⇒ switching to Coulomb distorted basis.

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Required: Computational implementation of Separable Optical Potential in Coulomb distorted basis in momentum space.

Separable Optical Nuclear Potential

Phenomenological optical potentials are usually in Woods-Saxon form in coordinate space.

Example: CH89 (central part)

$$U_{nucl}(r) = V(r) + i\big(W(r) + W_s(r)\big)$$

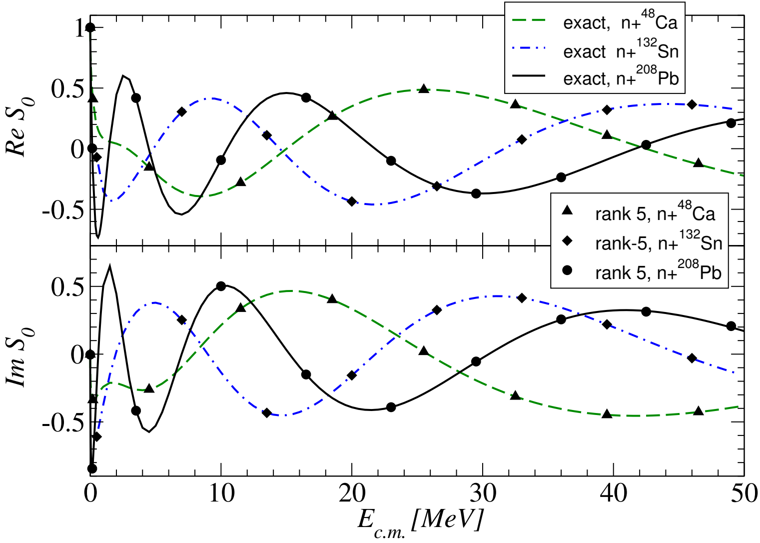
Separabilization: generalized Ernst-Shakin-Thaler scheme for complex optical potentials.

Now the form factors are not the arbitrary functions, but half-shell t-matrices.

$$U = \sum_{ij} u|\Psi_i^{(+)}\rangle\lambda\langle\Psi_j^{(-)}|u$$

Hint: In/Out states are necessary to fulfill reciprocity theorem.

Quality of Separable Optical Potential: $l = 0$, S-matrix



Half-shell t-martix in Coulomb basis

For complex potentials, Coulomb distorted half-shell t-matrices (form factors) are not the complex conjugate of one another:

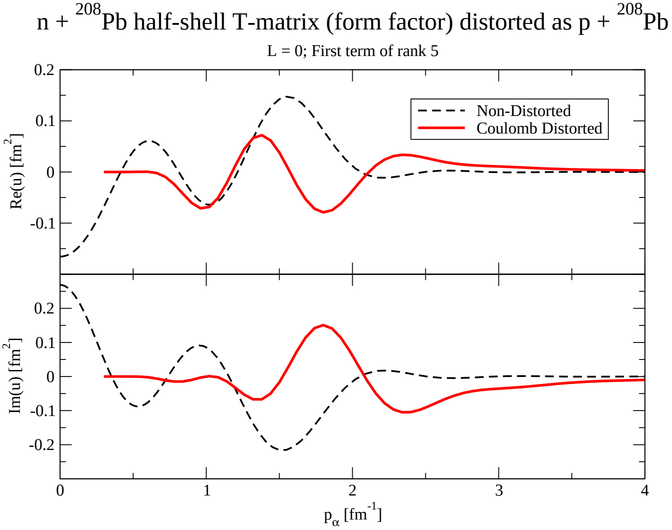
$$u_{li}^{(C,1)}(p_\alpha) = \frac{1}{2\pi^2} \int dp\, p^2 u_{li}(p) \psi_{p_\alpha l}^{C}(p);$$
$$u_{li}^{(C,2)}(p_\alpha) = \frac{1}{2\pi^2} \int dp\, p^2 u_{li}(p) [\psi_{p_\alpha l}^{C}(p)]^*;$$

$\psi_{p_\alpha l}^{C}(p)$ is the half-shell Coulomb scattering wave function for the asymptotic momentum p_α:

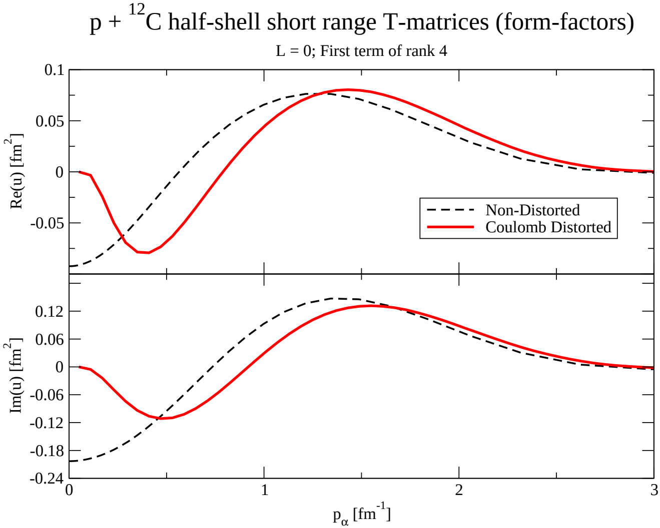
$$|\psi_{p_\alpha l}^{C}(p)\rangle = [1 + G_0(p_\alpha) T^C] |p\rangle.$$

- Special functions of complex arguments.
- Different representations for pole and non-pole regions.
- Gel'fand-Shilov regularization to deal with oscillatory singularity.

$$\left\{ u_{li}^{(C,1)}(p_\alpha) = \frac{1}{2\pi^2} \int dp\, p^2 u_{li}(p) \psi_{lp_\alpha}^{C}(p) \right\}$$



$$\left\{ u_{li}^{(C,1)}(p_\alpha) = \frac{1}{2\pi^2} \int dp\, p^2 u_{li}(p) \psi_{lp_\alpha}^{C}(p) \right\}$$



Summary & Outlook

Faddeev-AGS framework in Coulomb basis passed the first test!

- Momentum space nuclear form-factors (half-shell T-matrices) obtained in a Coulomb distorted basis for high charges for the first time.
- Ernst-Shakin-Thaler separabilization procedure successfully generalized for the case of complex optical potentials in momentum space. Realistic Separable (Generalized EST-type) Optical Potentials obtained for $n+{}^{12}\mathrm{C}$, ${}^{48}\mathrm{Ca}$, ${}^{123}\mathrm{Sn}$, and ${}^{208}\mathrm{Pb}$ cases.
- Algorithms to compute $\psi^{C}_{p_\alpha l}(p)$ and the overlap integral successfully implemented for Generalized EST-type optical potentials. "Oscillatory singularity" of $\psi^{C}_{p_\alpha l}(p)$ at $p = p_\alpha$ successfully regularized.

Near Future

Implementation of Faddeev-AGS equations in Coulomb basis to compute observables for (d, p) reactions.

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Half-shell t-matrix: Difficulties to be addressed

$$\psi^C_{p_\alpha l}(p) = -\frac{4\pi}{p} e^{-\pi\eta/2}\Gamma(1+i\eta)e^{i\alpha_l}\left[\frac{(p+p_\alpha)^2}{4pp_\alpha}\right]^l$$

$$\times \ \mathrm{Im}\left[e^{-i\alpha_l}\frac{(p+p_\alpha+i0)^{-1+i\eta}}{(p-p_\alpha+i0)^{1+i\eta}}\,{}_2F_1\left(-l,-l-i\eta;1-i\eta;\frac{(p-p_\alpha)^2}{(p+p_\alpha)^2}\right)\right];$$

$$\eta = Z_1 Z_2 e^2 \mu / p_\alpha.$$

- Computing special functions of complex arguments.
- Two different representations for pole and non-pole regions are required due to ${}_2F_1(a,b;c;z)$.
- $\psi^C_{p_\alpha l}(p)$ has 'oscillatory singularity' at $p = p_\alpha$.
  $\mapsto$ Gel'fand-Shilov regularization (reduce integrand around the pole, subtracting 2 terms of Taylor expansion).

Two representations of $\psi^C_{p_\alpha l}(p)$

Pole: $$\psi^C_{p_\alpha l}(p) = -\frac{4\pi}{p} e^{-\pi\eta/2}\Gamma(1+i\eta)e^{i\alpha_l}\left[\frac{(p+p_\alpha)^2}{4pp_\alpha}\right]^l$$
$$\times \operatorname{Im}\left[e^{-i\alpha_l}\frac{(p+p_\alpha+i0)^{-1+i\eta}}{(p-p_\alpha+i0)^{1+i\eta}}\,{}_2F_1\left(-l,-l-i\eta;1-i\eta;\zeta\equiv\frac{(p-p_\alpha)^2}{(p+p_\alpha)^2}\right)\right].$$

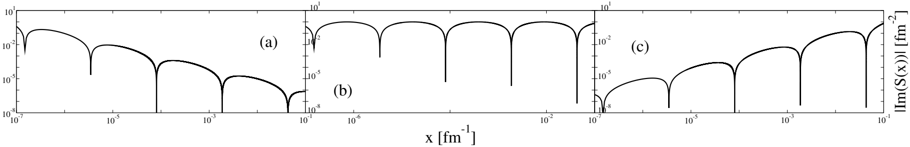
Switching point: $\zeta = \chi \approx 0.34.$ $\eta = Z_1 Z_2 e^2 \mu/p_\alpha.$

Non-Pole: $$\psi^C_{p_\alpha l}(p) = -\frac{4\pi\eta e^{-\pi\eta/2} p_\alpha (pp_\alpha)^2}{(p^2+p_\alpha^2)^{l+1+i\eta}}\left[\frac{\Gamma(l+1+i\eta)\Gamma(1/2)}{\Gamma(l+3/2)}\right]$$
$$\times [p^2-(p_\alpha+i0)^2]^{-1+i\eta}\,{}_2F_1\left(\frac{l+2+i\eta}{2},\frac{l+1+i\eta}{2};l+\frac{3}{2};\chi\equiv\frac{4p^2p_\alpha^2}{(p^2+p_\alpha^2)^2}\right)$$

Regularization

Gel'fand-Shilov regularization is the generalization of the Principal value regularization. The idea is to reduce the integrand $S(x)$ near the singularity[1,2]:

$$\int_{-\Delta}^{\Delta} dx \frac{\varphi(x)}{x^{1+i\eta}} = \int_{-\Delta}^{\Delta} dx \frac{\varphi(x) - \varphi(0) - \varphi'(0)x}{x^{1+i\eta}} - \frac{i\varphi(0)}{\eta}\left[\Delta^{-i\eta} - (\Delta)^{-i\eta}\right] + \dots$$



[1] This formula is significantly simplified.

[2] I. M. Gel'fand and G. E. Shilov. "Generalized Functions". Vol. 1. "Properties and Operations". Academic Press, New York and London. 1964.

Pinch singularity and avoiding it

Pinch

$\psi^C_{lp_\alpha}(k)$ has a singularity at $k = p_\alpha$. In general case of nuclear potential $V(p, p_\alpha)$,

$$(\psi^C_{lp}(k))^* \xrightarrow{k=p} V_{lpp_\alpha}(k,\kappa) \xleftarrow{\kappa=p} \psi^C_{lp_\alpha}(\kappa). \quad (1)$$

G. Cattappan et al. suggestion:

in case of separable potential, double integration procedure split onto two independent integrals, allowing to deal with this singularities separately, avoiding pinch[a].

[a] G. Cattappan et al. // Nucl. Phys. **A241** (1975) 204–218.