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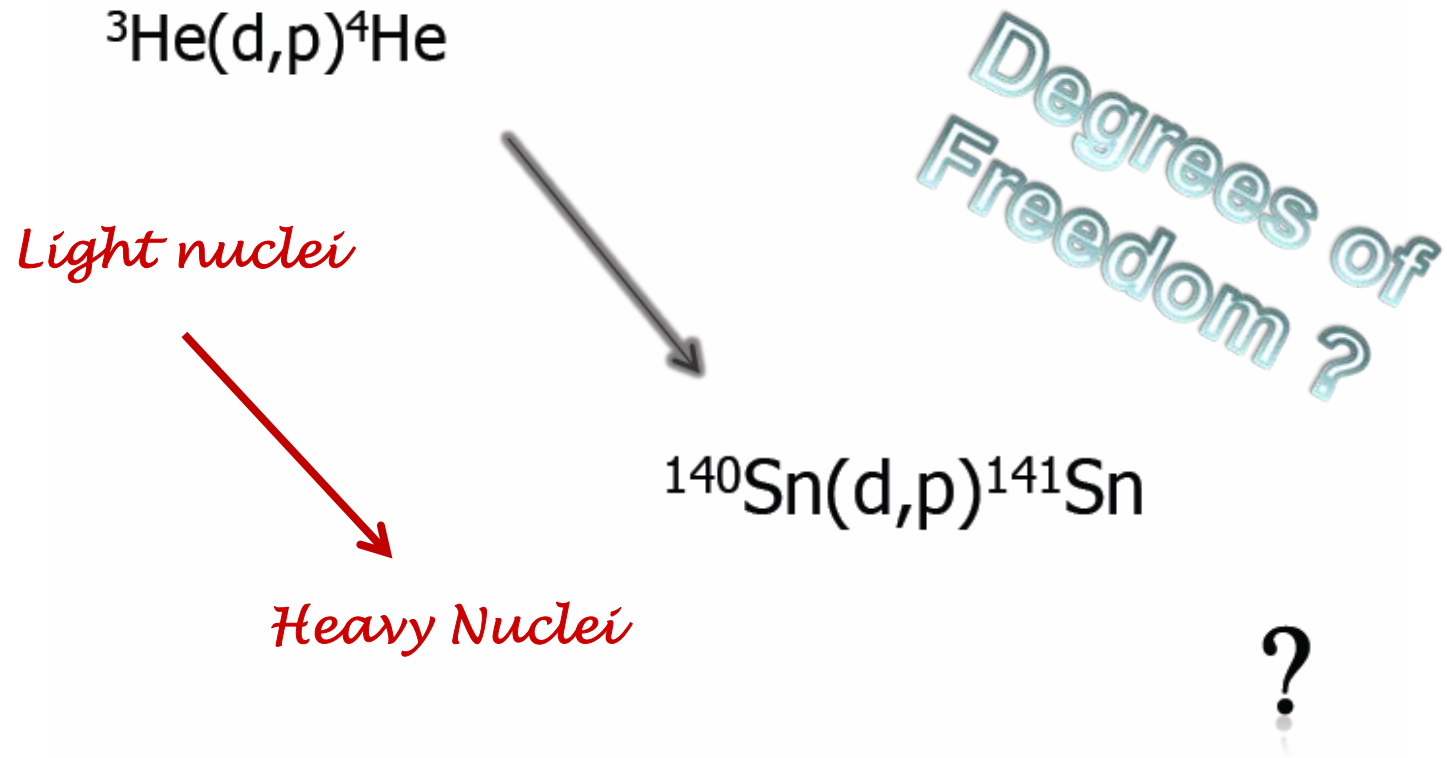
Momentum Space Coulomb Distorted Matrix Elements for Heavy Nuclei

Ch. Elster

N. Upadhyay, V. Eremenko, F. Nunes, L. Hlophe,
G. Arbanas, J. E. Escher, I.J. Thompson

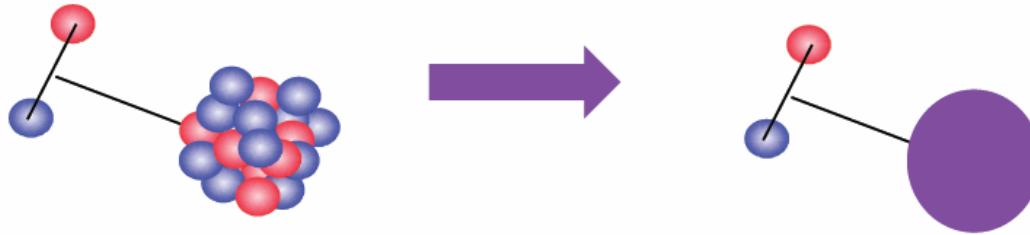
(The TORUS Collaboration)

What Reactions are we interested in?



Reactions: Elastic Scattering, Breakup, Transfer

Reduce Many-Body to Few-Body Problem



Task:

- Isolate important degrees of freedom in a reaction
- Keep track of important channels
- Connect back to the many-body problem

Hamiltonian for effective few-body problem:

$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

np interaction

Optical potentials p+A and n+A

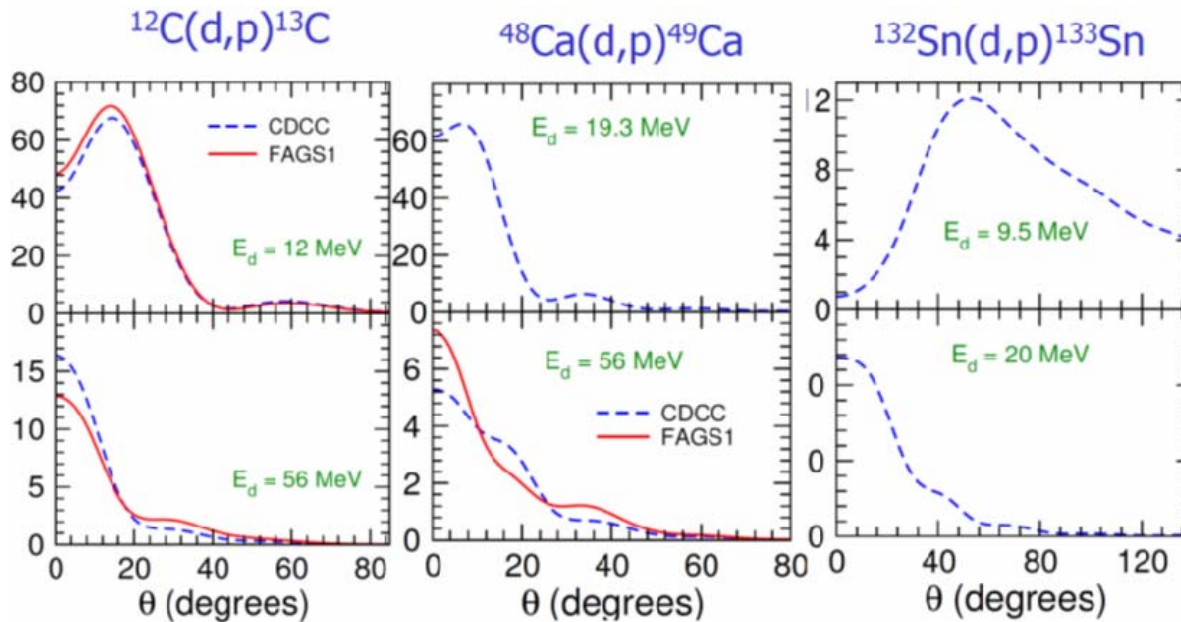
Three-Body Problem

(d,p) Reactions as three-body problem



Deltuva and Fonseca, Phys. Rev. C **79**, 014606 (2009).

Elastic, breakup, rearrangement channels are included and fully coupled (compared to e.g. CDCC calculations)



Issue: current momentum space implementation of Coulomb interaction (shielding) does **not** converge for $Z \geq 20$



A.M. Mukhamedzhanov, V.Eremenko and A.I. Sattarov,
Phys.Rev. C86 (2012) 034001

Solve Faddeev equations in Coulomb basis (no screening)

→ Implies integrals
like

$$Z_l^C(p, p_\alpha) = \int \frac{dp' p'^2}{2\pi} U_l(p, p') \psi_l^C, p_\alpha(p')$$

If $U_l(p, p') = \sum_{i,j} u_{l,i}^*(p) (M_l)_{i,j} u_{l,j}(p')$

Integral contains smooth function $u_{l,i}(p')$ and $\psi_{p_\alpha l}^C(p')$

Coulomb wave function in momentum space and pw decomposition

Very nasty! “pole” at $p_\alpha = p'$

Suggestion is new &
needs to be tested



First Test in Two-Body System



Calculate two-body Coulomb distorted nuclear matrix element

Separable nuclear Optical Potential

$$u_l(p'_\alpha, p_\alpha) = \sum_{ij} u_{li}^*(p'_\alpha) [M_l]_{ij} u_{lj}(p_\alpha)$$

$u_{li}(p_\alpha)$ is the nuclear potential form factor.

Compute: Coulomb distorted nuclear form factor

$$u_l^C(p_\alpha) = \frac{1}{2\pi^2} \int dp p^2 u_l(p) \psi_{p_\alpha l}^C(p)$$

$\psi_{p_\alpha l}^C(p)$ is the Coulomb scattering wave function

Challenges:



$$\psi_{p_\alpha l}^C(p) = -\frac{4\pi}{p} e^{-\pi\eta/2} \Gamma(1+i\eta) e^{i\alpha_l} \left[\frac{(p+p_\alpha)^2}{4pp_\alpha} \right]^l$$
$$\times \operatorname{Im} \left[e^{-i\alpha_l} \frac{(p+p_\alpha+i0)^{-1+i\eta}}{(p-p_\alpha+i0)^{1+i\eta}} {}_2F_1 \left(-l, -l-i\eta; 1-i\eta; \frac{(p-p_\alpha)^2}{(p+p_\alpha)^2} \right) \right]$$

$$\eta = Z_1 Z_2 e^2 \mu / p_\alpha.$$

- Compute special functions of complex arguments
- ${}_2F_1(a,b;c,z)$ requires two different representations for pole and non-pole regions
- **“oscillatory” singularity at $p = p_\alpha$**
- **Gel’fand-Shilov regularization**
 - Reduce integrand around pole by subtracting 2 terms of the Taylor series

Gel'fand-Shilov Regularization:

Generalization of Principal value regularization

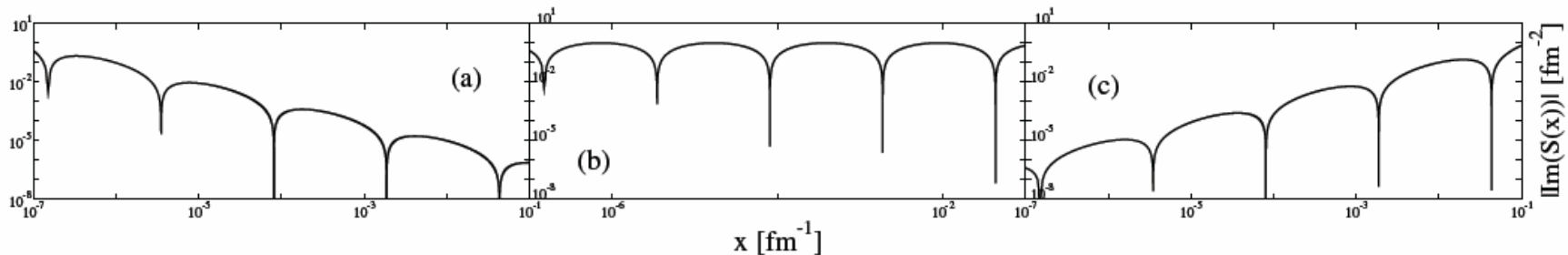
Idea: reduce value of integrand near singularity



$$\int_{-\Delta}^{\Delta} dx \frac{\varphi(x)}{x^{1+i\eta}} = \int_{-\Delta}^{\Delta} dx \frac{\varphi(x) - \varphi(0) - \varphi'(0)x}{x^{1+i\eta}}$$

simplified

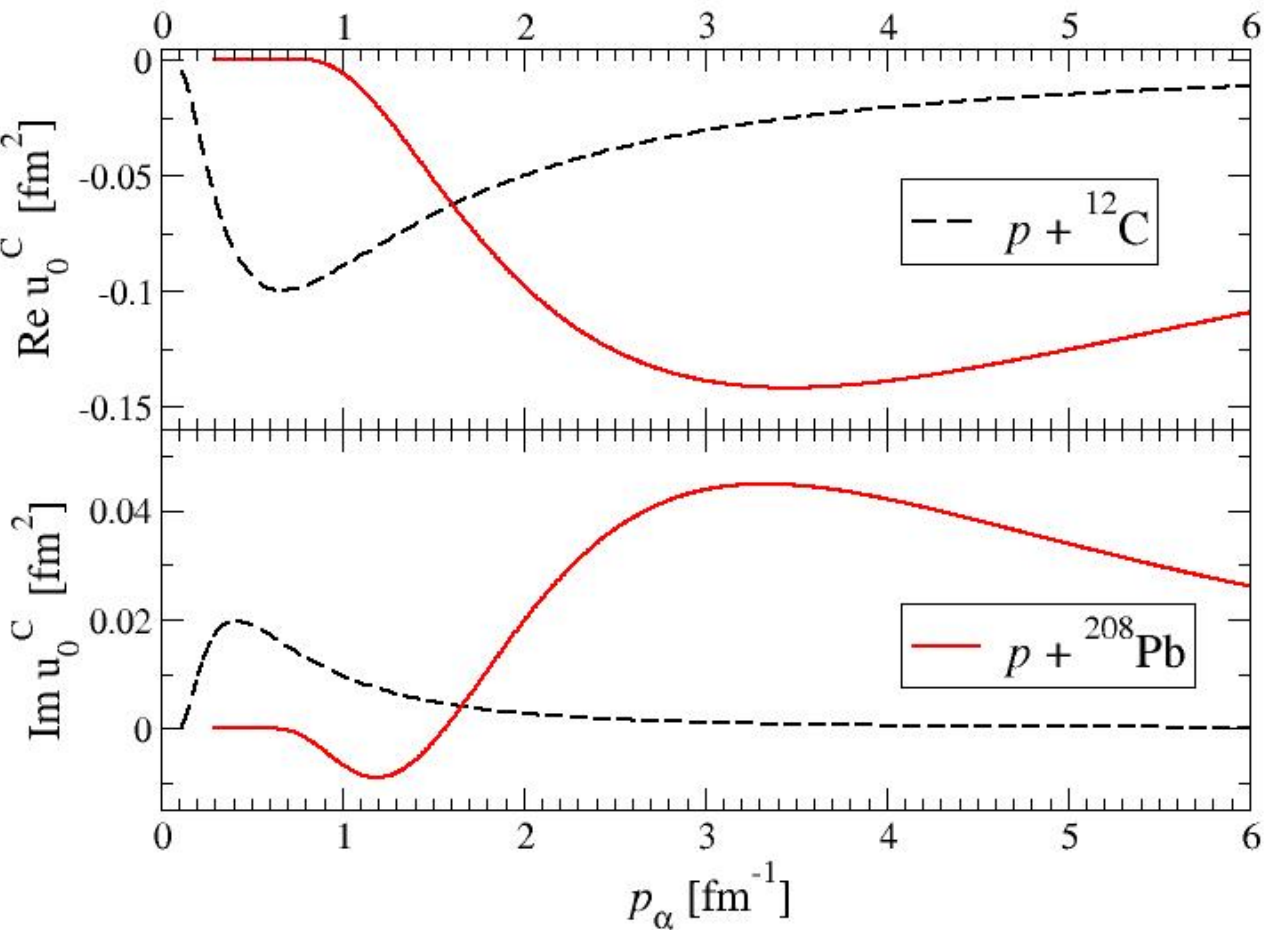
$$- \frac{i\varphi(0)}{\eta} [\Delta^{-i\eta} - (\Delta)^{-i\eta}] + \dots$$



I. M. Gel'fand and G. E. Shilov. "Generalized Functions". Vol. 1.

Academic Press, New York and London. 1964.

With Yamaguchi-type test form factor



**First calculation of
Coulomb distorted
 ^{208}Pb formfactor in
momentum space !**

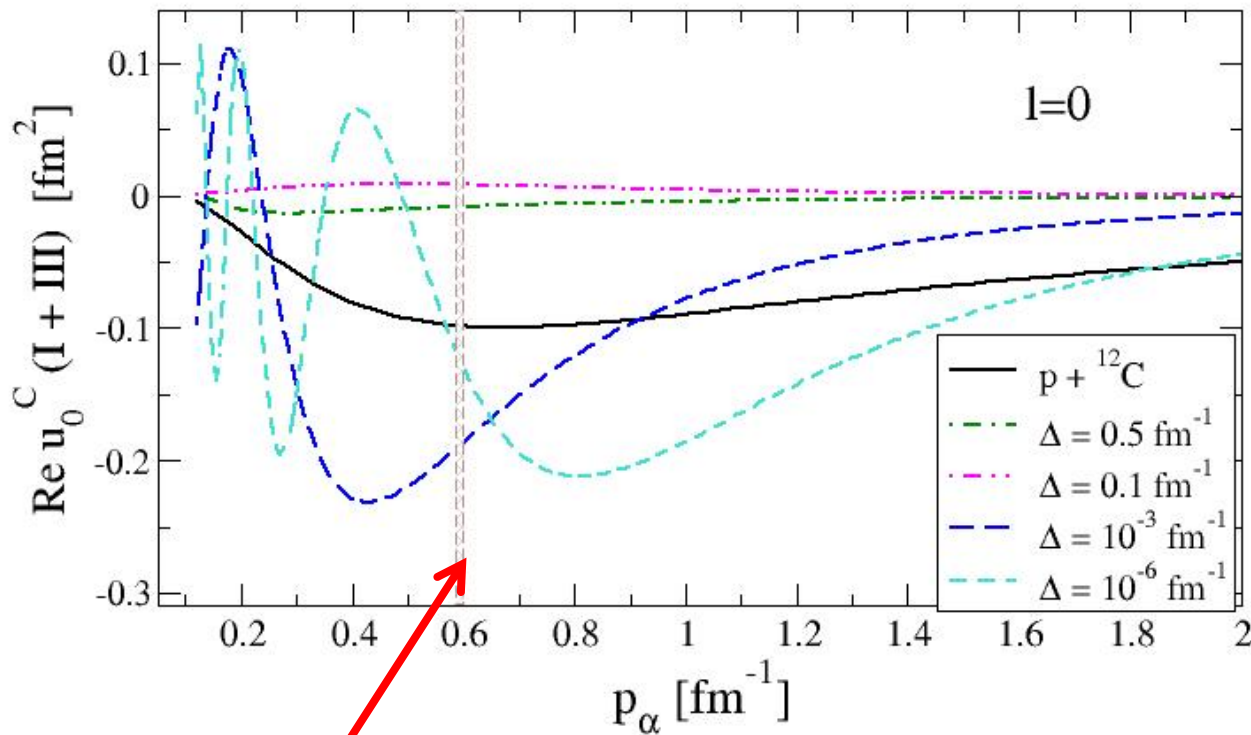
p + ¹²C



$$u_l^C(p_\alpha) = \underbrace{\int_0^{p_\alpha - \Delta} \frac{dp}{2\pi^2} p^2 u_l(p) \psi_{p_\alpha l}^C(p)}_I + \underbrace{\int_{p_\alpha - \Delta}^{p_\alpha + \Delta} \dots}_{II} + \underbrace{\int_{p_\alpha + \Delta}^\infty \dots}_{III}$$



Pole region

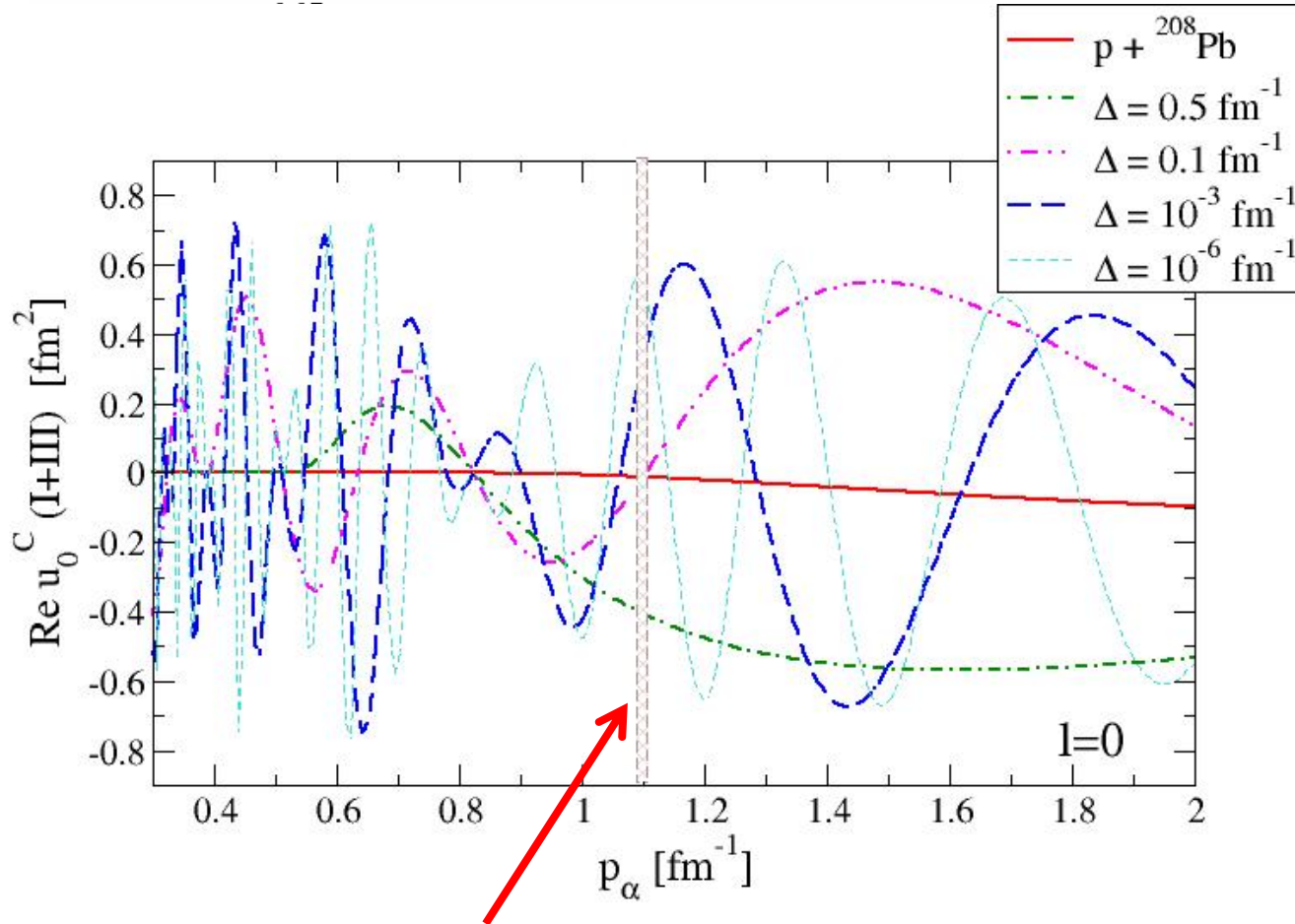


Fixed p_α

p + ²⁰⁸Pb

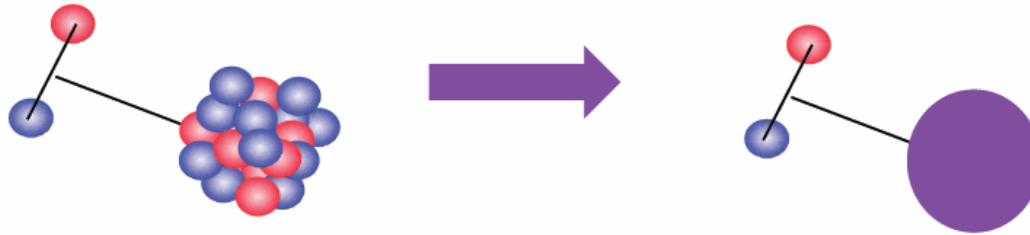


$$u_l^C(p_\alpha) = \underbrace{\int_0^{p_\alpha - \Delta} \frac{dp}{2\pi^2} p^2 u_l(p) \psi_{p_\alpha l}^C(p)}_I + \underbrace{\int_{p_\alpha - \Delta}^{p_\alpha + \Delta} \dots}_{II} + \underbrace{\int_{p_\alpha + \Delta}^{\infty} \dots}_{III}$$



Fixed p_α

Reduce Many-Body to Few-Body Problem



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np interaction

Optical potentials p+A and n+A

Three-Body Problem

Separable Representation of Optical Potentials

Starting point: Woods-Saxon Representation

Method of Ernst-Shakin-Thaler

BUT: Needs to be generalized for complex potentials

So that $\mathcal{K}U\mathcal{K}^{-1} = U^\dagger$ \mathcal{K} is the time-reversal operator.

$$U = \sum_{i,j} u |\Psi_i^{(+)}\rangle \langle \Psi_i^{(+)}| M |\Psi_j^{(-)}\rangle \langle \Psi_j^{(-)}| u$$

$$\delta_{ik} = \sum_j \langle \Psi_i^{(+)}| M |\Psi_j^{(-)}\rangle \langle \Psi_j^{(-)}| u |\Psi_k^{(+)}\rangle = \sum_j \langle \Psi_i^{(-)}| u |\Psi_j^{(+)}\rangle \langle \Psi_j^{(+)}| M |\Psi_k^{(-)}\rangle.$$

Definition with In/Out-states necessary to fulfill reciprocity theorem

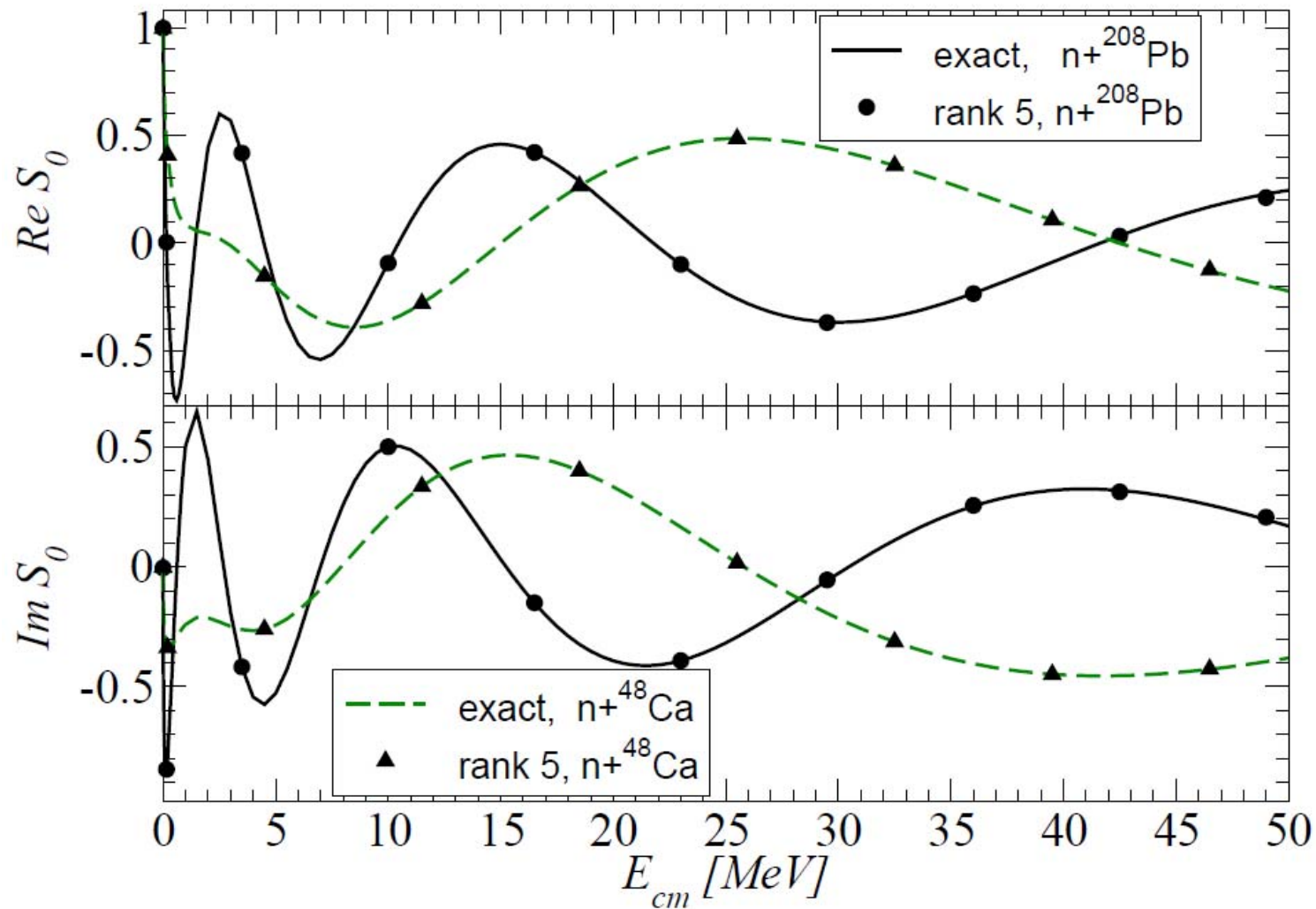
t-matrix:
$$t(E) = \sum_{i,j} u |\Psi_i^{(+)}\rangle \tau_{ij}(E) \langle \Psi_j^{(-)}| u$$

$$\sum_j \tau_{ij}(E) \langle \Psi_j^{(-)}| u - u g_0(E) u |\Psi_k^{(+)}\rangle = \delta_{ik}.$$

Compute and solve system of linear equations

$n + {}^{48}\text{Ca}$ and $n + {}^{208}\text{Pb}$: $l=0$

Chapel-Hill Optical Potential



Summary & Outlook

Faddeev-AGS framework in Coulomb basis passed first test!

- **Momentum space** nuclear form factors obtained in a Coulomb distorted basis for **high charges for the first time**.
- “Oscillatory singularity” of $\psi_{p_\alpha, l}^c(p)$ at $p = p_\alpha$ **successfully regularized**.
- Algorithms to compute $\psi_{p_\alpha, l}^c(p)$ and the overlap integral **successfully implemented**



In Progress:

Calculations with separable p+A optical potentials
(generalized EST scheme)

Near Future:

Implementation of Faddeev-AGS equations in the
Coulomb basis to obtain (d,p) observables



TORUS: Theory of Reactions for Unstable Isotopes

A Topical Collaboration for Nuclear Theory

<http://www.reactiontheory.org/>



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Some insights for momentum space Coulomb wave functions:

Pole:
$$\psi_{p_\alpha l}^C(p) = -\frac{4\pi}{p} e^{-\pi\eta/2} \Gamma(1+i\eta) e^{i\alpha_l} \left[\frac{(p+p_\alpha)^2}{4pp_\alpha} \right]^l$$

$$\times \text{Im} \left[e^{-i\alpha_l} \frac{(p+p_\alpha+i0)^{-1+i\eta}}{(p-p_\alpha+i0)^{1+i\eta}} {}_2F_1 \left(-l, -l-i\eta; 1-i\eta; \zeta \equiv \frac{(p-p_\alpha)^2}{(p+p_\alpha)^2} \right) \right]$$

Switching point: $\zeta = \chi \approx 0.34$

$$\eta = Z_1 Z_2 e^2 \mu / p_\alpha$$

Non-Pole:
$$\psi_{p_\alpha l}^C(p) = -\frac{4\pi\eta e^{-\pi\eta/2} p_\alpha (pp_\alpha)^2}{(p^2+p_\alpha^2)^{l+1+i\eta}} \left[\frac{\Gamma(l+1+i\eta)\Gamma(1/2)}{\Gamma(l+3/2)} \right]$$

$$\times [p^2 - (p_\alpha+i0)^2]^{-1+i\eta} {}_2F_1 \left(\frac{l+2+i\eta}{2}, \frac{l+1+i\eta}{2}; l+\frac{3}{2}; \chi \equiv \frac{4p^2 p_\alpha^2}{(p^2+p_\alpha^2)^2} \right)$$

**Some insights on momentum space Coulomb wave functions:
There are two representations for pole and non-pole regions**

$$\psi_{p_\alpha l}^C(p') = \frac{-4\pi e^{-\eta_\alpha \pi/2}}{p'} \left(\frac{(p' + p_\alpha)^2 + \gamma^2}{4p' p_\alpha} \right)^l \times \Gamma(1 + i\eta_\alpha) e^{i\alpha l}$$

$$\times \lim_{\gamma \rightarrow +0} \text{Im} \left\{ \left[e^{-i\alpha l} \frac{(p' + p_\alpha + i\gamma)^{i\eta_\alpha - 1}}{(p' - p_\alpha + i\gamma)^{i\eta_\alpha + 1}} \right. \right.$$

$$\left. \left. \times {}_2F_1 \left(-l, -l - i\eta_\alpha; 1 - i\eta_\alpha; \frac{(p' - p_\alpha)^2 + \gamma^2}{(p' + p_\alpha)^2 + \gamma^2} \right) \right] + \gamma \left[\dots \right] \right\}$$

$\psi_{p_\alpha l}^C(p')$ at low & high mc **Switch:** $\frac{4p'^2 p_\alpha^2}{(p'^2 + p_\alpha^2 + \gamma^2)^2} = \frac{(p' - p_\alpha)^2 + \gamma^2}{(p' + p_\alpha)^2 + \gamma^2}$

$$\psi_{p_\alpha l}^C(p') = -2\pi e^{-\eta_\alpha \pi/2} (p' p_\alpha)^l \left[\frac{\Gamma(l + 1 + i\eta_\alpha) \Gamma(\frac{1}{2})}{\Gamma(l + \frac{3}{2})} \right]$$

$$\times \lim_{\gamma \rightarrow +0} \left\{ \left[\left(\frac{2(p'^2 - (p_\alpha + i\gamma)^2)^{i\eta_\alpha}}{(p'^2 + p_\alpha^2 + \gamma^2)^{l+i\eta_\alpha+1}} \right) \left(\frac{\eta_\alpha(p_\alpha + i\gamma)}{p'^2 - (p_\alpha + i\gamma)^2} - \frac{\gamma(l + i\eta_\alpha + 1)}{p'^2 + p_\alpha^2 + \gamma^2} \right) \right. \right.$$

$$\left. \left. \times {}_2F_1 \left(\frac{l + i\eta_\alpha + 2}{2}, \frac{l + i\eta_\alpha + 1}{2}; l + \frac{3}{2}; \frac{4p'^2 p_\alpha^2}{(p'^2 + p_\alpha^2 + \gamma^2)^2} \right) \right] + \gamma \left[\dots \right] \right\}$$

Code will eventually be published

