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Coulomb Distorted nuclear matrix elements in momentum space
I. Formal Aspects

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(The TORUS Collaboration)

Department of physics + astronomy

Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

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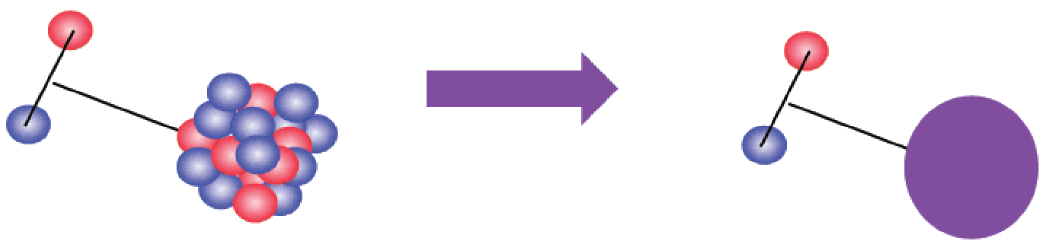
1804

(d,p) Reactions as tool for investigation nuclear structure

Reduce Many-Body to Few-Body Problem



Task:

- Isolate important degrees of freedom in a reaction
- Keep track of important channels
- Connect back to the many-body problem

Hamiltonian for effective few-body poblem:

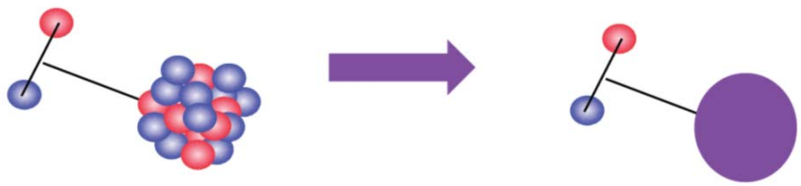
$$\mathbf{H = H_0 + V_{np} + V_{nA} + V_{pA}}$$

np interaction (→ V_{np})

Optical potentials p+A and n+A (→ $V_{nA} + V_{pA}$)

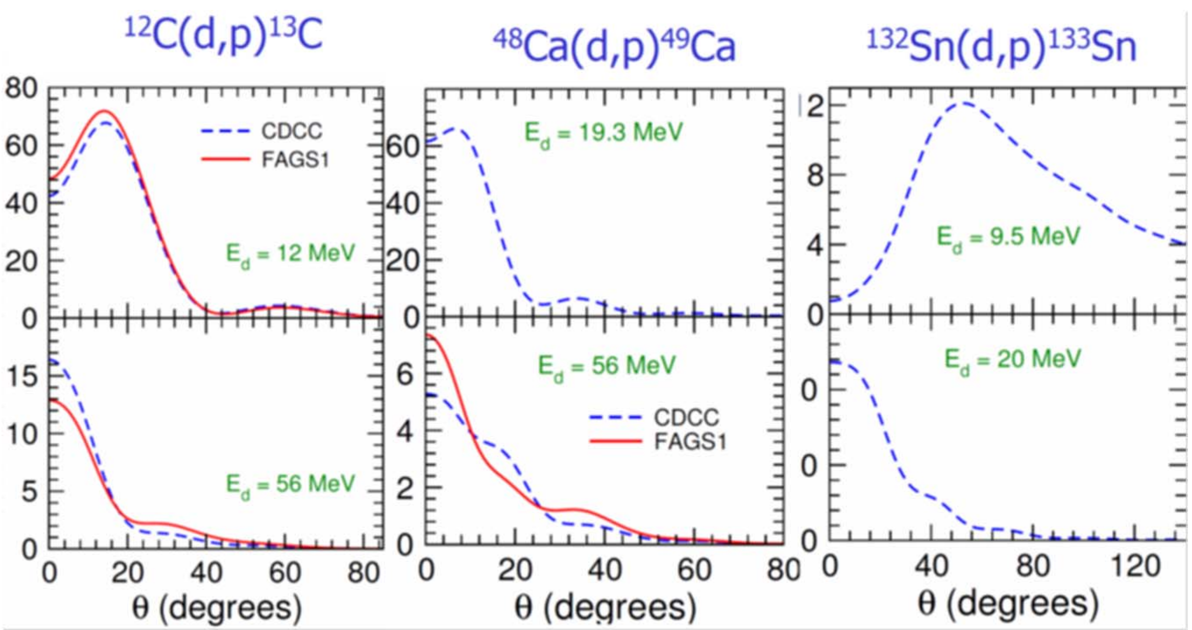
Three-Body Problem

(d,p) Reactions as three-body problem



Deltuva and Fonseca, Phys. Rev. C**79**, 014606 (2009).

Elastic, breakup, rearrangement channels are included and fully coupled (compared to e.g. CDCC calculations)



Issue: **current momentum space implementation of Coulomb interaction (shielding) does not converge for Z ≥ 20**

Courtesy: F.M. Nunes

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A.M. Mukhamedzhanov, V.Eremenko and A.I. Sattarov,
Phys.Rev. C86 (2012) 034001

Solve Faddeev equations in Coulomb basis (no screening)

→ Implies integrals like

$$Z_l^C(p, p_\alpha) = \int \frac{dp' p'^2}{2\pi} U_l(p, p') \psi_l^C, p_\alpha(p')$$

If $U_l(p, p') = \sum_{i,j} u_{l,i}^*(p) \, (M_l)_{i,j} \, u_{l,j}(p')$

Integral contains smooth function $u_{l,i}(p')$ and $\psi_{p_\alpha l}^C(p')$

Coulomb wave function in momentum space and pw decomposition

Very nasty! "pole" at $p_\alpha = p'$

Suggestion is new needs to be tested

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First Test in Two-Body System

Calculate two-body Coulomb distorted nuclear matrix element

Separable nuclear Optical Potential

$$u_l(p'_\alpha, p_\alpha) = \sum_{ij} u^*_{li}(p'_\alpha)\,[M_l]_{ij}\,u_{lj}(p_\alpha)$$

$u_{li}(p_\alpha)$ is the nuclear potential form factor.

***Compute:* Coulomb distorted nuclear form factor**

$$u_l^C(p_\alpha) = \frac{1}{2\pi^2}\int dp\, p^2 u_l(p)\,\psi^C_{p_\alpha l}(p)$$

$\psi^C_{p_\alpha l}(p)$ is the Coulomb scattering wave function

Challenges:

$$\psi^C_{p_\alpha l}(p) = -\frac{4\pi}{p} e^{-\pi\eta/2}\Gamma(1+i\eta)e^{i\alpha_l}\left[\frac{(p+p_\alpha)^2}{4pp_\alpha}\right]^l$$

$$\times\ \mathrm{Im}\left[e^{-i\alpha_l}\frac{(p+p_\alpha+i0)^{-1+i\eta}}{(p-p_\alpha+i0)^{1+i\eta}}\,{}_2F_1\left(-l,-l-i\eta;1-i\eta;\frac{(p-p_\alpha)^2}{(p+p_\alpha)^2}\right)\right]$$

$$\eta = Z_1 Z_2 e^2 \mu / p_\alpha.$$

- Compute special functions of complex arguments
- ${}_2F_1$ (a,b;c,z) requires two different representations for pole and non-pole regions
- **"oscillatory" singularity at $p = p_\alpha$**
- **Gel'fand-Shilov regularization**
  - Reduce integrand around pole by subtracting 2 terms of the Taylor series

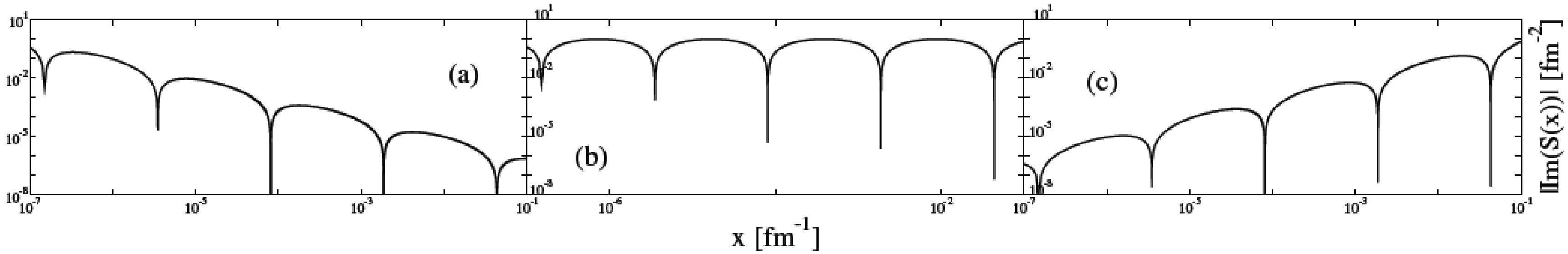
Gel'fand-Shilov Regularization:

Generalization of Principal value regularization

Idea: reduce value of integrand near singularity

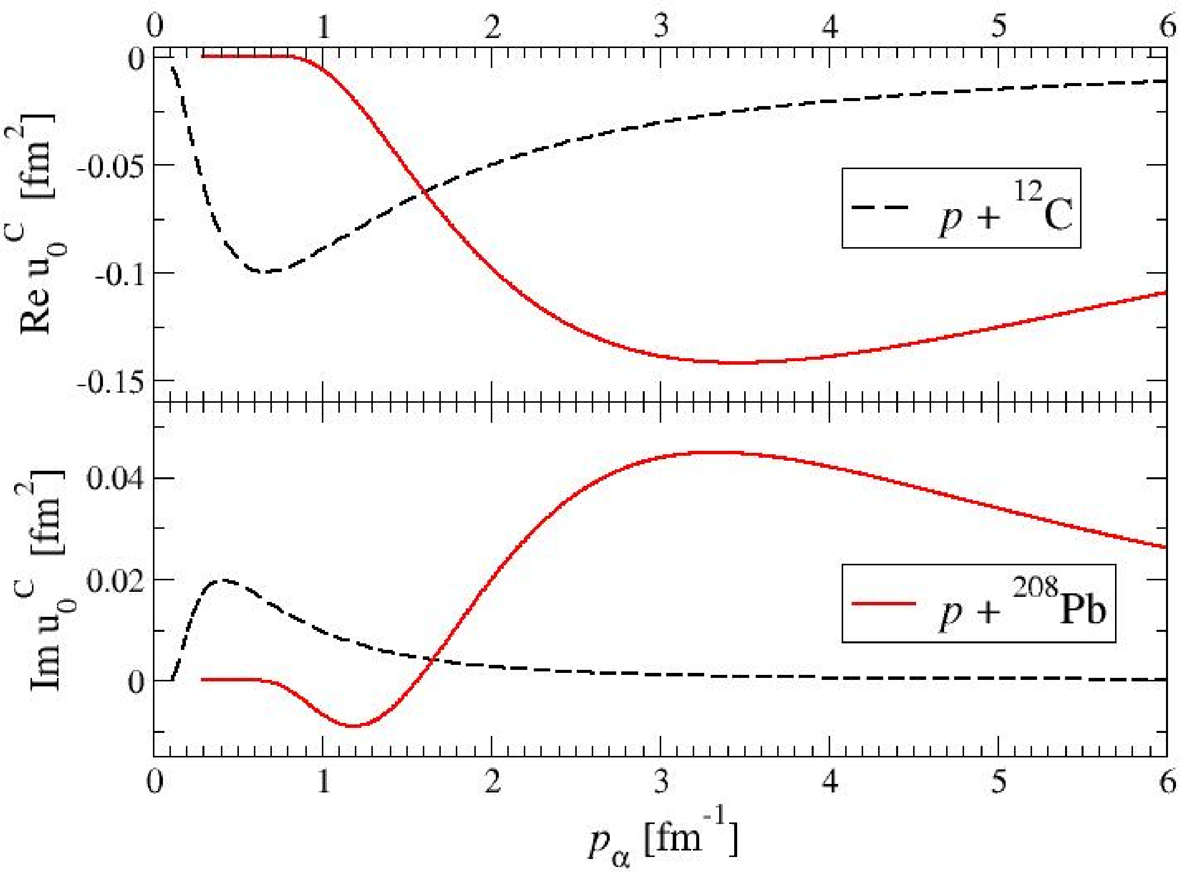
$$\int_{-\Delta}^{\Delta} dx \frac{\varphi(x)}{x^{1+i\eta}} = \int_{-\Delta}^{\Delta} dx \frac{\varphi(x) - \varphi(0) - \varphi'(0)x}{x^{1+i\eta}} - \frac{i\varphi(0)}{\eta}[\Delta^{-i\eta} - (\Delta)^{-i\eta}] + \ldots$$

simplified



I. M. Gel'fand and G. E. Shilov. "Generalized Functions". Vol. 1. Academic Press, New York and London. 1964.

With Yamaguchi-type test form factor



First calculation of Coulomb distorted ^{208}Pb formfactor in momentum space !

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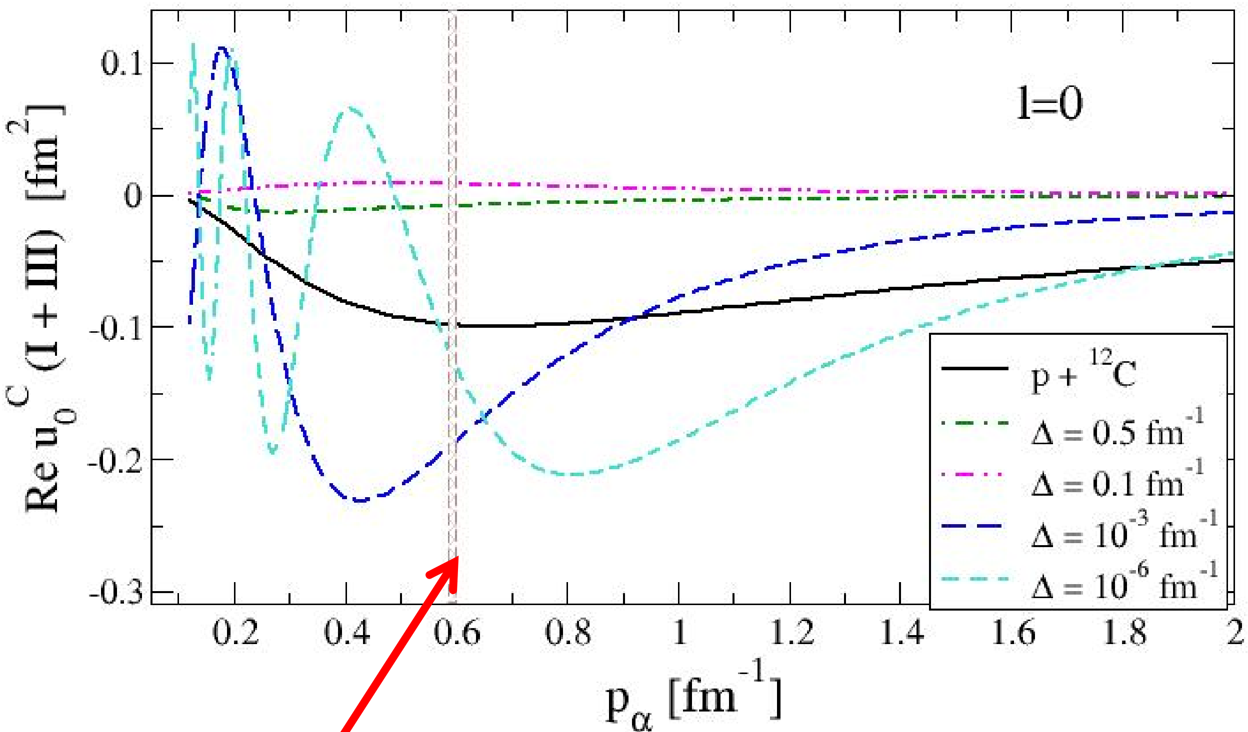
p + ^{12}C

$$u_l^C(p_\alpha) = \overbrace{\int_0^{p_\alpha-\Delta} \frac{dp}{2\pi^2}\, p^2 u_l(p)\, \psi^C_{p_\alpha l}(p)}^{I} + \overbrace{\int_{p_\alpha-\Delta}^{p_\alpha+\Delta} \ldots}^{II} + \overbrace{\int_{p_\alpha+\Delta}^{\infty} \ldots}^{III} .$$

Pole region



Fixed p_α

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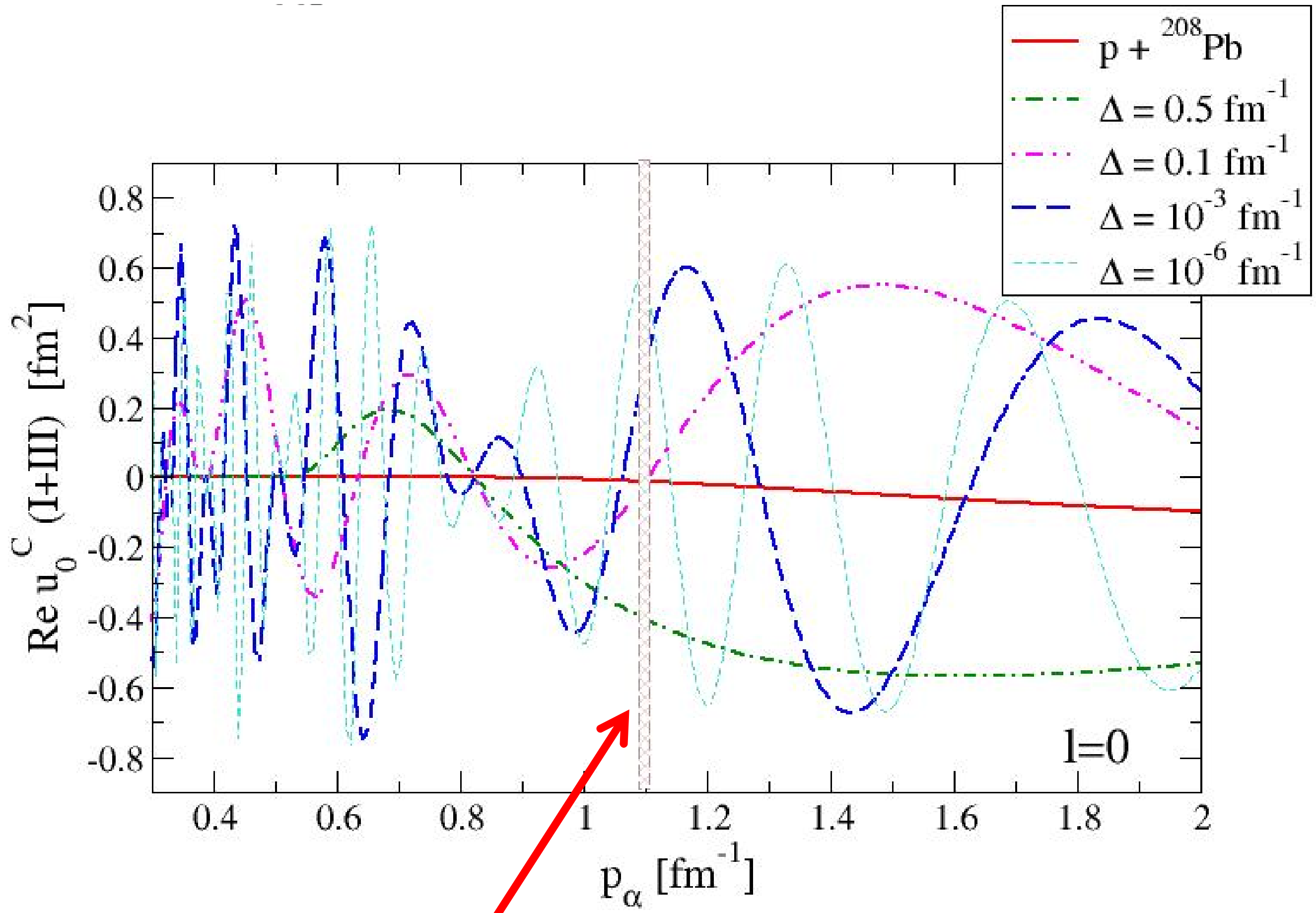
p + ^{208}Pb

$$u_l^C(p_\alpha) = \overbrace{\int_0^{p_\alpha-\Delta} \frac{dp}{2\pi^2}\, p^2 u_l(p) \psi^C_{p_\alpha l}(p)}^{I} + \overbrace{\int_{p_\alpha-\Delta}^{p_\alpha+\Delta} \ldots}^{II} + \overbrace{\int_{p_\alpha+\Delta}^{\infty} \ldots}^{III}.$$



Fixed p_α

Roadmap:

(d,p) Reactions as 3-Body Problem applicable for heavy (and light) nuclei

- **Formulation of Faddeev equations in Coulomb basis (no screening):**
  A.M. Mukhamedzanov, V. Eremenko, A.I. Sattarov (PRC 86 (2012) 034001)

- **Construction of separable optical potentials (n+^{12}C,^{48}Ca, ^{132}Sn, ^{208}Pb) :**
  L. Hlophe (Ohio U) and TORUS collaboration (manuscript ready)

- **Formulation of practical implementation of Coulomb distorted nuclear matrix elements with Yamaguchi test potential :**
  N. Uphadyay (MSU / LSU) and TORUS collaboration

- **Numerical implementation with realistic separable nuclear potential :**
  V. Eremenko (OU) and TORUS collaboration **(next talk)**

TORUS: Theory of Reactions for Ustable iSotopes

A Topical Collaboration for Nuclear Theory

http://www.reactiontheory.org/

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