

Adiabatic approximation versus exact Faddeev method for (d, p) and (p, d) reactionsF. M. Nunes^{1,2} and A. Deltuva³¹*National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824, USA*²*Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824-1321*³*Centro de Física Nuclear, Universidade de Lisboa, P-1649-003 Lisboa, Portugal*

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The finite-range adiabatic wave approximation (ADWA) provides a practical method to analyze (d, p) or (p, d) reactions; however, until now the level of accuracy obtained in the description of the reaction dynamics has not been determined. In this work, we perform a systematic comparison between the finite-range adiabatic wave approximation and the exact Faddeev method. We include studies of $^{11}\text{Be}(p, d)^{10}\text{Be}(\text{g.s.})$ at $E_p = 5, 10$, and 35 MeV; $^{12}\text{C}(d, p)^{13}\text{C}(\text{g.s.})$ at $E_d = 7, 12$, and 56 MeV; and $^{48}\text{Ca}(d, p)^{49}\text{Ca}(\text{g.s.})$ at $E_d = 19, 56$, and 100 MeV. Results show that the two methods agree within $\approx 5\%$ for a range of beam energies ($E_d \approx 20\text{--}40$ MeV) but differences increase significantly for very low energies and for the highest energies. Our tests show that ADWA agrees best with the Faddeev method when the angular momentum transfer is small $\Delta l = 0$ and when the neutron-nucleus system is loosely bound.

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I. INTRODUCTION

There is growing interest in using (d, p) reactions as a tool to extract single-particle properties of nuclei away from stability. Since the early studies on ^{11}Be [1], an increasing number of transfer experiments have been performed to obtain detailed structure information. This includes experiments at the ISOL facilities, performed around the Coulomb barrier, such as the recent one on ^{132}Sn [2], but also measurements at higher energy in fragmentation facilities, such as the recent systematics on Ar isotopes [3]. Because the information extracted from these experiments relies on reaction models, it is critical to validate these models and assess their uncertainties.

There are a number of aspects that need to be considered when validating reaction theories. At present, reaction theories for (d, p) , applicable to nuclei with mass $A > 10$, require a reduction of the many-body problem to a few-body problem. A consequence of this first point is the introduction of effective interactions between the composite constituents, the so-called optical potentials, and single-particle binding potential. These are the physical inputs to the problem. In addition, there are often approximations in solving the few-body problem. It is this last aspect that is the focus of the present work.

For 40 years, the tradition has been to use the distorted wave Born approximation (DWBA) to extract spectroscopic information from (d, p) or (p, d) reactions [4]. Even though still in use, this method has been challenged repeatedly. Just in the last decade, a variety of reaction theory studies have been carried out. In Ref. [5], it is shown that the choice of the deuteron optical potential in the DWBA introduces a very large ambiguity which can be significantly reduced when using the zero-range adiabatic wave approximation developed by Johnson and Soper (ZR-ADWA) [6] built on the nucleon-target optical potentials. In the ZR-ADWA deuteron breakup is taken into account to all orders, while making a zero-range approximation for the deuteron. The formalism for a finite-range version of the adiabatic wave method (FR-ADWA) was developed in Ref. [7], and is also built on nucleon optical

potentials. Perhaps even larger than the uncertainties in optical potentials, the ambiguity introduced by the single-particle potentials that describes the many-body overlap function has also been a focus of study [8,9]. Target excitation is known to be relevant for specific cases and should be considered case by case [9,10].

To add to the long list of reaction theories, one should still consider the continuum-discretized coupled-channel (CDCC) method [11] and the Faddeev-type method for transition operators, usually referred to as the Alt-Grassberger-Sandhas (AGS) method [12]. Although the CDCC method is probably most widely used to describe breakup (e.g., Ref. [13]), the CDCC wave function has also been used in the context of (d, p) reactions (e.g., Refs. [14,15]). The AGS method including the Coulomb interaction, originally developed for describing few-nucleon reactions [16], has recently been extended to handle (d, p) and (p, d) nuclear reactions [17]. The CDCC method takes breakup effects into account to all orders without making any further approximations (in some ways, the ADWA is an approximate version of the CDCC method), but the Faddeev method goes a step further in that transfer channels are also included explicitly in the expansion. The Faddeev calculations represent thus the exact solution to the full three-body problem when rearrangement channels are present. While both CDCC and AGS methods make less approximations compared to the DWBA and the ADWA, they are computationally demanding. Moreover, their technical implementations are limited, for example, the present treatment of Coulomb in the AGS method has been successfully applied only to nuclei with charge $Z < 30$ [18]. Finally, and most importantly, these two methods are based on a more complex expansion of the wave function, in such a way that the final cross section depends not only on the many-body overlap of interest but also on nontrivial interferences between many overlap functions including overlaps with states in the continuum. As a consequence, simpler approximate methods are often preferred by experimentalists.

Recently, a systematic study [19] using the finite-range version of the adiabatic wave approximation (FR-ADWA) [7,20] found large finite-range effects in (d, p) reactions, especially at the higher energies. The transfer cross section in the FR-ADWA depends on the overlap function of interest, and not on a complicated superposition of terms with many overlap functions. Also important, the method is not computationally expensive and is of practical use for the nonexpert. Nevertheless, one needs to appreciate that the FR-ADWA relies on a Sturmian expansion that is truncated to first order (first order should not be confused with the DWBA, because the adiabatic method relies on an entirely different expansion). This first-order truncation of the exact solution of the three-body problem has never been systematically tested before. FR-ADWA calculations in Ref. [21] are used to analyze^{34,36,46} Ar(p, d) data at 33 MeV/u and compared to the Faddeev calculations to determine the error in the treatment of the reaction dynamics. Discrepancies between the FR-ADWAs were found to vary considerably (6–19%). The work in Ref. [21] calls for a better understanding of the range of validity of the FR-ADWA.

The aim of the present work is exactly to determine the range of validity of the ADWA (we drop FR from now on, since all ADWA calculations presented here are finite range). In order to do that, a systematic comparison between ADWA and Faddeev calculations is performed. We cover a wide range of beam energies, light- and intermediate-mass nuclei, and well-bound and loosely bound systems. We choose energies for which there are data to ensure that the reaction theory performs sensibly, although our focus is on comparing two theories starting from the same three-body Hamiltonian, not on dwelling on detailed comparisons with data. In Sec. II, we provide some key aspects of the ADWA and Faddeev methods, and in Sec. III results are presented and discussed. Finally in Sec. IV conclusions are drawn.

II. THEORETICAL DESCRIPTION

The starting point for both the ADWA and Faddeev methods is the three-body Hamiltonian for $n + p + A$:

$$\mathcal{H}_{3B} = T_{\mathbf{r}} + T_{\mathbf{R}} + U_{nA} + U_{pA} + V_{np}. \quad (1)$$

The interactions between the nucleons and the composite target A (U_{pA} and U_{nA}) should contain an imaginary term representing the absorption or transfer to other channels not explicitly included. We focus the discussion on the reaction $A(d, p)B$ to the ground state (g.s.) of $B = n + A$ (note that the formalism for (p, d) in prior form is identical to that for (d, p) in post form presented here [15]).

Following the work on the ZR-ADWA [6], Johnson and Tandy [7] introduced a full finite-range version considering the square-integrable Sturmian expansion. The Sturmian basis is complete within the range of the interaction and is defined by

$$(T_r + \alpha_i V_{np})S_i(\vec{r}) = -\varepsilon_d S_i(\vec{r}) \quad (2)$$

for each given state corresponding to the number of nodes $i = 0, 1, 2, \dots$. The inner product between Sturmian states is

defined by $\langle S_i | V_{np} | S_j \rangle = -\delta_{ij}$, and at large distances, all basis states decay exponentially according to the deuteron binding energy. The three-body wave function is now expanded in terms of S_i :

$$\Psi^{(+)}(\vec{r}, \vec{R}) = \sum_{i=0}^{\infty} S_i(\vec{r}) \chi_i(\vec{R}). \quad (3)$$

This form for the three-body wave function is then introduced in the three-body Schrödinger equation for scattering $H_{3B}\Psi = E\Psi$ which, when imposing the appropriate boundary conditions, allows one to calculate $\chi_i(\vec{R})$. This procedure leads to a nontrivial coupled-channel equation [7,19] that could be solved exactly [22].

Simplicity is recovered when considering only the first term in the expansion equation (3):

$$\Psi_{AD}^{(+)}(\vec{r}, \vec{R}) = S_0(\vec{r}) \chi_0^{AD}(\vec{R}). \quad (4)$$

Then the coupled-channel equation reduces to an optical-model-type equation with distorting potential,

$$U_{AD}(R) = -\langle S_0(\vec{r}) | V_{np}(U_{nA} + U_{pA}) | S_0(\vec{r}) \rangle, \quad (5)$$

where, apart from a normalization factor, S_0 is the ground-state wave function of the deuteron.

The three-body wave function equation (4) is now inserted in the post-form T matrix for the (d, p) process [15]:

$$T = \langle \phi_{nA} \chi_{pB}^{(-)} | V_{np} + \Delta_{\text{rem}} | \Psi_{AD}^{(+)} \rangle, \quad (6)$$

where $\chi_{pB}^{(-)}$ is the outgoing proton wave, distorted by U_{pB}^* ; ϕ_{nA} is the wave function of the final bound state, generated by a binding potential V_{nA} ; and the remnant term is $\Delta_{\text{rem}} = U_{pA} - U_{pB}$.

Even if we were not truncating the Sturmian basis, one would expect the method to be at its best when remnant contributions are small, because then the resulting cross section is only sensitive to short distances between the neutron and the proton, where the basis is complete. However, it is not clear that this condition is sufficient when the additional simplification, Eq. (4), is introduced.

As opposed to the ADWA, in the Faddeev approach, elastic scattering, transfer, and breakup channels are treated on equal footing. Therefore, it represents the exact solution to the problem once a three-body Hamiltonian equation (1) is defined. In the Faddeev method, the wave function is expanded in an overcomplete basis, involving all three Jacobi coordinates [4]. In the AGS method [12], one starts from the Faddeev formalism and arrives at coupled integral equations for the transition operators

$$T_{\beta\alpha} = (1 - \delta_{\beta\alpha})G_0^{-1} + \sum_{\gamma=1}^3 (1 - \delta_{\beta\gamma})t_{\gamma}G_0 T_{\gamma\alpha}, \quad (7)$$

whose on-shell matrix elements $\langle \psi_{\beta} | T_{\beta\alpha} | \psi_{\alpha} \rangle$ are scattering amplitudes and therefore lead directly to the observables. In Eq. (7), $G_0 = (E + i0 - H_0)^{-1}$ is the free resolvent, with E being the available three-particle energy in the center of mass (c.m.) system and H_0 the free Hamiltonian. The two-particle transition matrix is a solution of the Lippmann-Schwinger

equation

$$t_\gamma = v_\gamma + v_\gamma G_0 t_\gamma, \quad (8)$$

where v_γ is the potential for the pair γ ; we use the odd-man-out notation. The channel states $|\psi_\gamma\rangle$ for $\gamma = 1, 2, 3$ are the eigenstates of the corresponding channel Hamiltonian $H_\gamma = H_0 + v_\gamma$ with the energy eigenvalue E ; thus, $|\psi_\gamma\rangle$ is a product of the bound-state wave function for pair γ and a plane wave with fixed on-shell momentum corresponding to the relative motion of particle γ and pair γ in the initial or final state.

The Coulomb interaction is an additional complication in the AGS method; nevertheless, when only two particles are charged and charge numbers Z are not too large, the Coulomb interaction has been successfully included [17,18] using the method of screening and renormalization [23,24].

We solve the AGS equations in the momentum-space partial-wave framework [25]. The Coulomb screening radius needed for the convergence of the results increases with Z , thereby increasing also the number of partial waves that have to be included. Such calculations are not only time-consuming but also for very high angular momenta the partial-wave expansion may even become unstable. This limits the application of the technique with respect to Z ; so far no calculations have been performed beyond $Z = 30$ [18]. Another limitation of the method comes from the Pade summation technique for solving the AGS integral equation iteratively [26]; usually it is hard to achieve convergence for heavier nuclei at low energies. For more details on the numerical techniques we refer the reader to Refs. [23,25–27].

III. RESULTS

We perform Faddeev AGS and finite-range ADWA calculations for a number of cases that have been repeatedly studied: $^{11}\text{Be}(p,d)^{10}\text{Be}(\text{g.s.})$, $^{12}\text{C}(d,p)^{13}\text{C}(\text{g.s.})$, and $^{48}\text{Ca}(d,p)^{49}\text{Ca}(\text{g.s.})$. For the ADWA calculations, the code [32] is used to calculate the deuteron adiabatic potential, and the code [33] is used to calculate the transfer amplitude and cross sections. Because we are interested in establishing the range of validity of the ADWA, we explore these at several beam energies. For nearly all cases we choose beam energies for which data are available and we make sure that the present theories are able to provide a fair description of the angular distributions. However, since in this work we are concerned with testing the ADWA against the exact Faddeev AGS theory, we do not include data in the plots.

One of the main inputs to these calculations is the pair interactions. In both calculations, a realistic NN interaction is used. Because the ADWA is performed in coordinate space, and a local form of the NN interaction with spin-orbit coupling is needed, the Reid interaction [28] is used. The more recent momentum-based CD-Bonn [29] is straightforward to use in AGS calculations, contrary to potentials in coordinate space. These two NN interactions reproduce the low-energy NN phase shifts, the deuteron binding energy, and the quadrupole moment. In fact, apart from the short distances, the deuteron densities produced with Reid and CD-Bonn are identical, this being true also for the $3N$ and $4N$ systems. We have verified that the use of different NN interactions does not induce

differences in the predicted (d,p) calculations by repeating both Faddeev and ADWA calculations with a simple Gaussian interaction, which reproduces low-energy phase shifts and the deuteron binding energy. We concluded that the transfer cross sections are not sensitive to the details of this interaction but the consistency between the V_{np} used to obtain the deuteron wave function and the interaction in the transfer operator [Eq. (6)]. This same conclusion was found in previous works [17].

For the nucleon optical potentials we use the global parametrization CH89 [30]. Like all global optical potentials, the parameters of CH89 are functions of the beam energy. In the ADWA, when determining Ψ_{AD} in Eq. (6), half the deuteron energy is used for U_{nA} and U_{pA} , whereas the exit proton energy is used for the auxiliary potential U_{pB} (for calculating χ_{pB} and the remnant contribution). The same U_{pA} used for Ψ_{AD} is used in the transfer operator. A real interaction V_{nA} is used to calculate the bound state ϕ_{nA} : it consists of a Woods-Saxon potential plus spin-orbit form with the Woods-Saxon depth adjusted to reproduce the experimental binding energy of the nucleus under study. In all cases we fix the geometry of the Woods-Saxon potential, the radius $r = 1.25$ fm and the diffuseness $a = 0.65$ fm, as well as the spin-orbit parameters $V_{\text{so}} = 6$ MeV, $r_{\text{so}} = 1.25$ fm, and $a_{\text{so}} = 0.65$ fm. Thus, strictly speaking, since the neutron interaction is different in the incoming and outgoing channels, two different three-body Hamiltonians are used in the ADWA.

In the Faddeev calculation, energy dependence in the interactions can introduce orthogonality errors. It is thus important to make a wise choice for the energy at which U_{pA} is to be calculated. Our standard choice is to fix U_{pA} at the energy in the proton channel (FADD); however, we also show results using half the deuteron energy (FADD2). Concerning the neutron-target interaction, for the partial wave where a bound state exists, the same V_{nA} interaction is used as for the ADWA. For all other partial waves, the neutron interaction is U_{nA} , taken from CH89 at half the deuteron energy [30]. In our standard Faddeev calculation (FADD), there are two subtle differences in the interactions as compared to the ADWA: (i) U_{pA} for the deuteron channel is obtained at the proton energy, and (ii) for the partial waves where a neutron-bound state exists, the neutron scattering potential U_{nA} has no absorption. Because we have seen a larger dependence on the transfer cross sections to U_{pA} than to U_{nA} in previous calculations (see, e.g., Fig. 10 of Ref. [31]), we believe the uncertainty due to (i) dominates the overall uncertainty.

In Fig. 1 we present the angular distribution for $^{11}\text{Be}(p,d)^{10}\text{Be}$, at $E_p = 5$ MeV, $E_p = 10$ MeV, and $E_p = 35$ MeV. The blue (solid) lines are the results obtained with the ADWA and the red curves are those obtained with Faddeev calculations. To understand the subtle difference in the interactions included in the Faddeev calculations compared to the ADWA, we perform two test calculations: (i) a Faddeev calculation using the proton optical potential calculated at half the deuteron energy (red open circles labeled FADD2) and (ii) an ADWA calculation where the initial proton distorted wave is calculated with an optical potential determined at half the deuteron energy (purple solid squares labeled ADWA2). The comparison between FADD and FADD2 provides an estimate of the sensitivity to the choice of energy at which the

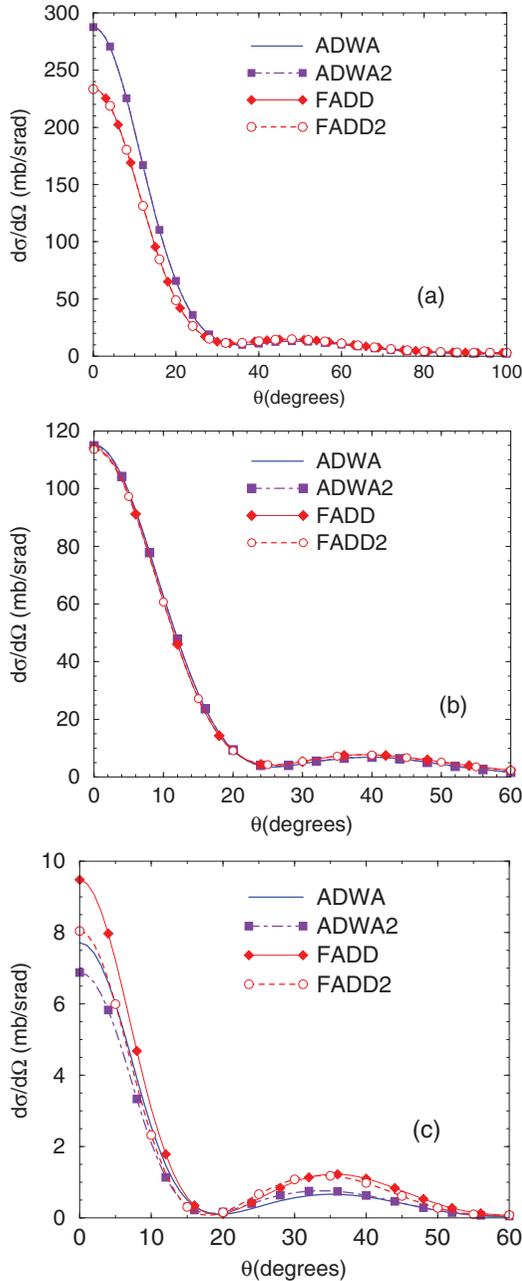


FIG. 1. (Color online) Angular distributions for $^{11}\text{Be}(p,d)^{10}\text{Be}$: (a) $E_p = 5$ MeV, (b) $E_p = 10$ MeV, and (c) $E_p = 35$ MeV.

proton-target optical potential is evaluated. If the two methods were exact, the comparison between FADD2 and ADWA2 would provide an estimate of the effect of replacing, in the calculation of Ψ_{AD} , the appropriate neutron optical potential by a real bound-state potential in the partial wave where a bound state exists. Of course ADWA2 is not exact, and therefore the comparison of FADD2 and ADWA2 contains both the difference due to the neutron optical potential (which should be small) and a genuine difference between the treatment of the three-body dynamics. Note that if Ψ_{AD} were exact, the T matrix of Eq. (6) should provide cross sections independent of the auxiliary potential U_{pB} (used in χ_{pB} and Δ_{rem}). This was recently demonstrated within the CDCC method [15].

Therefore, differences between ADWA and ADWA2 indicate inaccuracies in ADWA.

First of all, there are differences between Faddeev and ADWA cross sections at the lower and higher energies. At $E_p = 10$ MeV the agreement is perfect. Comparing FADD with FADD2, we find that only at the higher energy is there a dependence in the choice of energy at which the proton interaction is calculated, and then Faddeev calculations using the proton interaction at half the deuteron energy become slightly closer to the corresponding ADWA results (compare FADD2 and ADWA2). At $E_p = 35$ MeV, ADWA and ADWA2 differ by 11% at the peak, which indicates that the truncation to the first term in ADWA is insufficient at these energies.

In Fig. 2 the same observable is shown for $^{12}\text{C}(d,p)^{13}\text{C}$ at $E_d = 7.15$ MeV, $E_d = 12$ MeV, and $E_d = 56$ MeV. Differences between the Faddeev and ADWA calculations are smallest at $E_d = 12$ MeV. As for ^{11}Be , only at the highest beam energy are the results sensitive to the choice of energy at which the proton optical potential is defined. In this case, calculating the final proton distorted wave with U_{pB} evaluated at half the deuteron energy slightly improves the agreement (compare ADWA2 versus FADD2). This is not to say that adiabatic calculations for (d,p) should be performed using U_{pB} at half the deuteron energy. The standard ADWA makes the correct choices. Here ADWA2 only helps to understand which part of the disagreement with Faddeev calculations comes from the slight differences in the proton interaction.

Our last case, $^{48}\text{Ca}(d,p)^{49}\text{Ca}$, is shown in Fig. 3. Angular distributions were calculated at $E_d = 19.3$ MeV, $E_d = 56$ MeV, and $E_d = 100$ MeV. The comparison between the ADWA and Faddeev results follows the same trend as in the previous cases: good agreement at around 10 MeV/u, with large deterioration at very high energies. It was not possible to obtain Faddeev solutions for energies lower than 19.3 MeV due to the lack of convergence of the Pade summation technique. Oddly, for the reactions at 56 MeV, while the ADWA results show a very strong dependence on the choice of U_{pB} , the Faddeev results only exhibit a dependence on the proton optical potential around the second peak. At 100 MeV, the angular distribution is extremely sensitive to the choice of the proton energy used to determine the proton optical potential parameters for both ADWA and FADD. This introduces large ambiguity in the comparison of the ADWA and the Faddeev approach at 100 MeV.

Overall, we find the differences between the ADWA and Faddeev results to be below 10% except for the lowest and highest energies. Our results suggest that there is an optimum beam energy (around 10–20 MeV/u) where the ADWA is at its best, differing from the Faddeev calculation by only a few percent. Apart from the 100 MeV $^{48}\text{Ca}(d,p)$, changes introduced in the shape of the angular distributions by the adiabatic approximation are small. A quantitative summary of our results, plus additional detail in our calculations, is given in Table I. Here Δ_{F-AD} is the percentage difference between cross sections from FADD and ADWA, relative to the FADD cross section determined at the first peak of the distribution (corresponding to θ). To study the effect of the choice of the energy at which the proton interaction is determined, we also show Δ_{F-F2} , the percentage difference

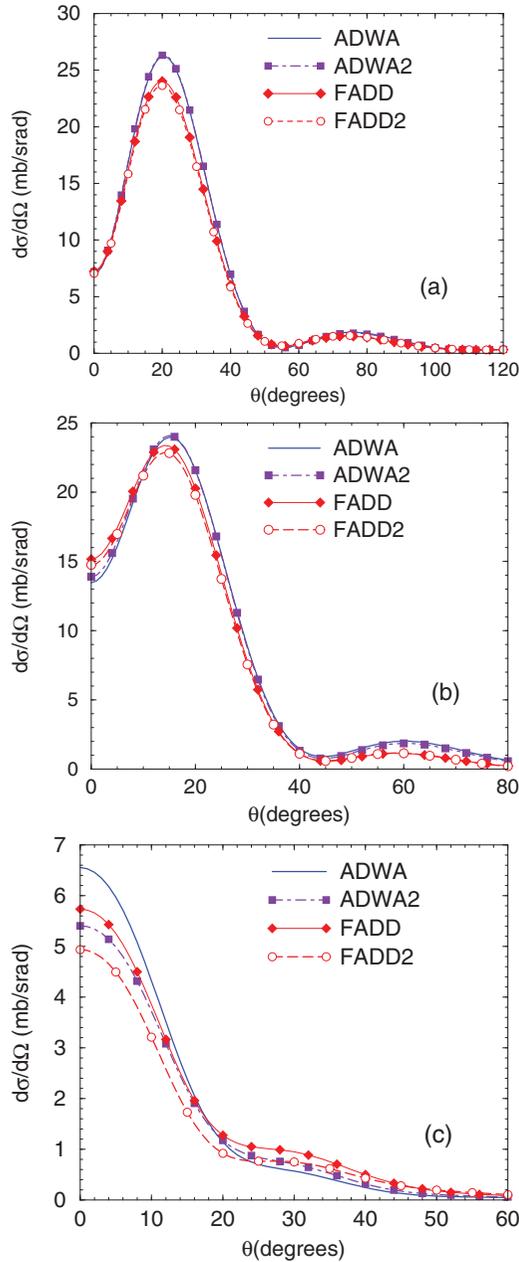


FIG. 2. (Color online) Angular distributions for $^{12}\text{C}(d, p)^{13}\text{C}$: (a) $E_d = 7.15$ MeV, (b) $E_d = 12$ MeV, and (c) $E_d = 56$ MeV.

between Faddeev calculations using the energy in the proton channel to determine U_{pA} (FADD) and those using half the deuteron energy (FADD2). This represents the ambiguity in the three-body Hamiltonian for the Faddeev calculations and can be used as an uncertainty in the comparison. It is not important for low and intermediate beam energies, and therefore in this beam-energy region, the agreement between Faddeev and ADWA calculations is robust. For completeness, the identical quantity for ADWA $\Delta_{\text{AD-AD2}}$ is also shown (where the standard ADWA is compared with that where the proton distorted wave is calculated with CH89 using half the deuteron energy ADWA2). Because the exact T matrix should be independent of the choice of the auxiliary

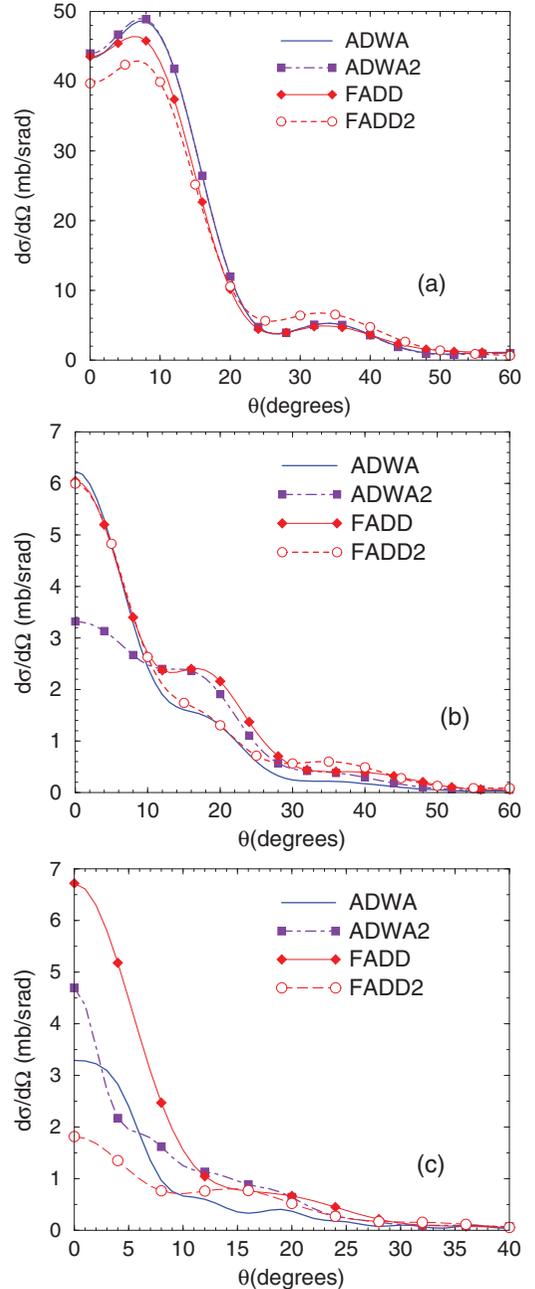


FIG. 3. (Color online) Angular distributions for $^{48}\text{Ca}(d, p)^{49}\text{Ca}$: (a) $E_d = 19.3$ MeV, (b) $E_d = 56$ MeV, and (c) $E_d = 100$ MeV.

potential, $\Delta_{\text{AD-AD2}}$ provides an internal warning sign that the approximation in the ADWA is not adequate. Consistent with the comparison with Faddeev calculations, the effect becomes significant for the higher beam energies.

In Sec. II, we pointed out that the Sturmian expansion is expected to be good in the region of V_{np} but not for large neutron-proton distances. For this reason, we do not expect the ADWA to work when the contribution of the remnant term in Eq. (6) is large. The last column of Table I contains the percentage difference between the original ADWA and the ADWA calculation neglecting the remnant term, relative to the original ADWA. The only case for which

TABLE I. Percentage differences between the differential cross section at the first peak of the distribution for the various formulations: Δ_{F-AD} , comparing the Faddeev approach with the adiabatic model; Δ_{F-U_p} , the effect of changing the energy at which the proton-target interaction is calculated in the Faddeev approach; and finally Δ_{AD-rem} , the effect of the remnant term in the adiabatic model. Also given is the beam energy E (in MeV) and the angle θ at which the percentage difference was calculated (in degrees).

Reaction	E	θ	Δ_{F-AD}	Δ_{F-F2}	Δ_{AD-AD2}	Δ_{AD-rem}
$^{11}\text{Be}(p,d)$	5	1	-22.90	0.17	-0.10	1.36
	10	1	-1.12	0.42	0.44	-1.13
	35	1	18.50	15.25	10.9	26.82
$^{12}\text{C}(d,p)$	7	20	-9.08	1.61	-0.42	-3.74
	12	15	-2.74	2.18	-0.50	-3.92
	56	1	-14.26	13.95	17.5	3.34
$^{48}\text{Ca}(d,p)$	19	8	-6.12	7.16	-0.64	-0.74
	56	1	-3.05	0.76	46.4	0.57
	100	0	51.0	72.9	-42.7	9.73

the remnant contribution is large is for the high-energy reaction with ^{11}Be and then, indeed, the ADWA performs poorly in comparison with the Faddeev calculation. However in the other cases, it is not the remnant contribution that is responsible for the disagreement ADWA versus Faddeev. Since the largest disagreement comes at the highest beam energies, this suggests that the main source for the ADWA disagreement with the Faddeev method comes from the truncation of the Sturmian expansion, Eq. (2). Equation (4) implies that all $\chi_i^{AD}(R)$ behave as the elastic $\chi_0^{AD}(R)$. However, at higher beam energies, large excitation energies in the deuteron channel are expected to affect the shape of the adiabatic distorted waves $\chi_i^{AD}(R)$.

An attempt to systematize the results is done in Fig. 4 where the ratio of Faddeev cross sections at the first peak of the angular distribution (θ from Table I) over the corresponding ADWA cross sections is plotted as a function of $E_d^{c.m.}/V_c$, the c.m. energy of the deuteron ($E_d^{c.m.}$) over a simple estimate of the Coulomb barrier $V_c = 1.44Z/(1.2A^{1/3})$. We introduce an error bar corresponding to the ambiguity in the choice of the energy at which the proton interaction is calculated in the Faddeev method (second column in Table I). Figure 4 provides a good illustration of the validity of the ADWA for intermediate energies. For the lowest energies, differences increase up to 20% and they become particularly large for the highest energies where the Hamiltonian ambiguities are also the largest.

In addition there should also be an error due to the fact that in the Faddeev approach, the n -target interaction in the deuteron channel has no absorption for the partial wave in which there exists a bound state. Based on previous experience, we expect this effect to be smaller than the effect on the choice of U_{pA} [31]. Additional Faddeev calculations for the lighter targets, introducing energy dependence in the n -target interaction in the partial wave corresponding to the bound state, suggest the ambiguity on U_{nA} is only comparable to Δ_{F-F2} for $^{11}\text{Be}(p,d)$ at 35 MeV.

We have also explored the dependence on the angular momentum l of the neutron-bound state as well as its binding

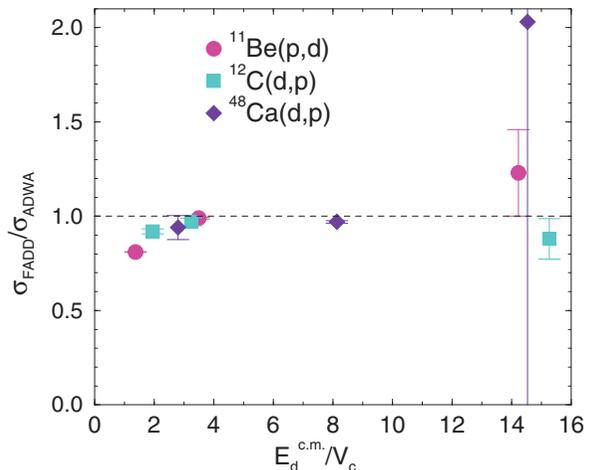


FIG. 4. (Color online) Ratio of the Faddeev prediction for the cross section at the first peak of the angular distribution versus the adiabatic counterpart plotted in terms of the deuteron energy in the c.m. over the Coulomb barrier.

energy $-S_n$. For this purpose, we repeat the calculations for our best case, $^{11}\text{Be}(p,d)^{10}\text{Be}$ at 10 MeV. The neutron-bound state in ^{11}Be has $l = 0$ and $S_n = 0.5$ MeV. We first repeat the calculations changing the angular momentum to $l = 1, 2, 3$ by adjusting the depth of the neutron-bound state interaction V_{nA} to reproduce the binding $S_n = 0.5$ MeV, while keeping all other interactions fixed. We find that as l increases, the difference between FADD and ADWA also increases: $\Delta(F - AD) = 1\%$ for $l = 1$; $\Delta(F - AD) = 15\%$ for $l = 2$; and $\Delta(F - AD) = 30\%$ for $l = 3$.

Next we repeat the calculations fixing $l = 0$ and changing the depth of V_{nA} to reproduce $S_n = 2.0, 4.0$ and 6.0 MeV. Again, all other interactions are unchanged. The percentage difference relative to the Faddeev result is as follows: $\Delta(F - AD) = 1\%$ for $S_n = 0.5$ MeV, $\Delta(F - AD) = 13\%$ for $S_n = 2.0$ MeV, $\Delta(F - AD) = 18\%$ for $S_n = 4.0$ MeV, and $\Delta(F - AD) = 23\%$ for $S_n = 6$ MeV. In these calculations, remnant effects do not change significantly. However, the deeper the neutron initial state, the more delocalized is the n - p system in the continuum, and one might expect the first term of the Sturmian expansion $\chi_0^{AD}(R)$ to have the wrong radial behavior. Whereas with increasing l the ADWA diverges from the Faddeev result in a rapid manner; the differences with increasing binding appear to reach a saturation point around $\approx 20\%$. These results explain the findings in Ref. [21]. In Ref. [21], the discrepancy between the ADWA and Faddeev models was found to be much larger for $^{36}\text{Ar}(p,d)$ than for $^{34}\text{Ar}(p,d)$ and $^{46}\text{Ar}(p,d)$. The hole state created by pulling out a neutron from ^{36}Ar has both, large l as well as large S_n .

IV. SUMMARY AND CONCLUSIONS

The finite-range adiabatic model provides a practical method to analyze (d,p) and (p,d) reactions that goes well beyond the DWBA. One of the great advantages is that it uses nucleon optical potentials, rather than deuteron optical potentials, which are far more ambiguous. However, it is by no

means the exact solution and in this work we quantify the errors introduced by comparing it to exact Faddeev AGS calculations. We choose three emblematic reactions. $^{11}\text{Be}(p,d)^{10}\text{Be}(\text{g.s.})$, $^{12}\text{C}(d,p)^{13}\text{C}(\text{g.s.})$, and $^{48}\text{Ca}(d,p)^{49}\text{Ca}(\text{g.s.})$, and span a large range of beam energies. Overall, we find the agreement for reactions with beam energies around 10 MeV/u to be better than 10%. These results bear important implications for the reactions being measured at the Isotope-Separator-on-Line facilities as well as for the science program at the future Facility for Rare Isotope Beams. The deviation of the ADWA from the Faddeev model increases with the angular momentum of the neutron-bound state as well as with the separation energy of the neutron. The dependence on l was found to be stronger than that on the separation energy.

The comparison with the Faddeev model is limited in several ways. On one hand there are limitations with fixing the three-body Hamiltonian. On the other hand there are technical issues that limit the number of cases that can be studied. To address the first point, we have explored in detail the effect of different choices for the proton-target interaction, namely, on the proton energy used to determine the optical potential. The differences obtained are then used as a systematic uncertainty in the comparison. They are only important at the higher energy, exactly where the performance of the ADWA is at its worse. In this sense, the conclusion that the ADWA fails for energies larger than $E_d^{\text{c.m.}}/V_c \approx 12$ is not robust.

As mentioned in Sec. II, there are several technical limitations in the present implementation of the Faddeev equations, namely, the AGS method in momentum space. In this respect we found $^{48}\text{Ca}(d,p)$ at 19.3 MeV to be most challenging and close to the limit of present capabilities in obtaining converged results.

Because the finite-range adiabatic model for $(d,p)/(p,d)$ at intermediate energies appears to be well suited to describe the few-body reaction dynamics for many cases of interest, is easy to use, and is not computationally intensive, it is strongly desirable that the standard reaction codes incorporate this option in a user-friendly manner.

In this work we did not consider target excitation; however, it is understood that there will be many applications in which target excitation is an integral part of the reaction mechanism. It would thus be worthwhile to extend the ADWA to include the inelastic channels in the formulation consistently.

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