

Exploring R-matrix ideas for the description of one-nucleon transfer reactions

J E Escher¹, A M Mukhamedzhanov² and I J Thompson¹

¹Lawrence Livermore National Laboratory, Livermore, CA 94551, U.S.A.

²Cyclotron Institute, Texas A&M University, College Station, TX 77843, U.S.A.

E-mail: escher1@llnl.gov

Abstract. Deuteron-induced reactions, in particular (d,p) one-neutron transfer reactions, have been used for decades to investigate the structure of nuclei. These reactions, carried out in inverse kinematics, are expected to play a central role in the study of weakly-bound systems at modern radioactive beam facilities. While the theoretical framework and its computational implementation for describing (d,p) reactions have seen much progress over the decades, open questions remain and need to be addressed, including the proper treatment of transfers to resonance states. Recently, a new formalism was proposed [1] that, in principle, describes transfers to both bound and resonance states. The new formalism is summarized here and illustrated; implications are discussed.

1. Introduction

The (d,p) reaction provides the simplest mechanism for creating a more neutron-rich isotope in specific final states by adding a neutron to a target nucleus. The reaction has been extensively used to study the properties of single-particle orbitals: their spins and parities, their energies and evolutions with neutron number. With the move to radioactive beam facilities, it becomes possible to study a much wider range of isotopes, in particular those short-lived species that will provide new insights into the nuclear force and that play an important role in astrophysical environments. As experimental techniques have been greatly improved over the decades, the theoretical descriptions of the reactions have also evolved. Butler's simple theory has made way to distorted-wave descriptions, coupled-channels treatments have been introduced to account for processes beyond the one-step mechanisms, and state-of-the-art approaches include contributions from the breakup of the deuteron [2, 3, 4]. More work is needed, though, to ensure that proper techniques are available and tested for applications in the new neutron and proton-rich regimes of interest.

An area in need of improvement is the treatment of reactions that populate resonance states. Resonances, long-lived but unstable states, occur in light, medium, and heavy nuclear systems. Their properties, in particular their energies and widths, are of great interest to nuclear astrophysics, as many reaction that occur in astrophysical environments proceed through resonances. Furthermore, resonances provide both challenges and stringent tests for nuclear structure models, as predicting their properties involves a proper treatment of the nuclear many-body system including the continuum. Current theories of transfer reactions that populate resonance states suffer from two major shortcomings. On the practical side, one has to deal

with convergence issues, as the matrix element that describes the transition to the final state includes contributions from very large distances outside the nucleus. Conceptually, it is also not clear what spectroscopic information can be extracted from a transfer to a resonance; a solid connection between experimental observables and typical resonance parameters remains to be established. Recent formal work by Mukhamedzhanov [1] has laid the foundation for going beyond these limitations. In Ref. [1], cross sections for (d,p) transfer reactions are expressed in terms of parameters that are typically employed in R-matrix theory. R-matrix theory is a standard tool for extracting resonance properties such as energies and widths from nucleon capture and scattering experiments [5, 6], and the new approach establishes a similar link between resonance properties and transfer reactions.

In the next section, we briefly summarize the formalism developed in Ref. [1], in particular the introduction of ‘exterior’ and ‘interior’ contributions to the transfer matrix element and the emergence of a surface term. In Section 3, we will illustrate the underlying ideas with examples and discuss the implications.

2. R-matrix concepts in binary and transfer reactions

Experimentally, resonance structures are most often studied in elastic and inelastic scattering reactions. For almost all nuclei resonance properties cannot be theoretically predicted, but the phenomenological R-matrix approach has been extremely useful for the interpretation of experiments and for extracting resonance energies and widths from measured cross sections [5, 6]. An extension of the approach, applicable to (d,p) transfer reactions, was recently developed [1] with the goal to provide a practical and sound way for extracting structure information from transfer experiments.

2.1. R-matrix approach to binary reactions

In the R-matrix approach for a binary reaction, the configuration space is separated into two regions: an exterior region, in which the reaction partners are well-separated and interact only via the Coulomb interaction (for charged particles), and an interior region, where a confined compound system exists that is governed by the nuclear and Coulomb forces. Formally, the nuclear wave function in the interior is expanded in some suitable set of basis functions, while in the exterior, it takes the form of a scattering function, and matching conditions are imposed at the surface. In typical applications, the parameters are adjusted to reproduce measured cross sections. This approach makes it possible to parameterize the collision matrix, and thus the calculated cross section, in terms of a few formal R-matrix parameters, which can then be related to observed quantities.

2.2. R-matrix ideas in the description of (d,p) reactions

Even though R-matrix theory [5] is typically employed in the description of binary reactions, such as elastic or inelastic scattering, or capture reactions, the underlying ideas can be applied more generally. For (d,p) reactions, an extension of the approach was worked out by Mukhamedzhanov [1], who introduced a surface that defines interior and exterior regions. The separation into the different regions is based on the distance r_{nA} of the deposited neutron from the center of the target nucleus, in analogy to what is done in descriptions of binary reactions. The standard distorted-wave Born Approximation (DWBA) transition matrix element can be written in post or prior form:

$$M^{(post)} = \langle \varphi_F \chi_{pF}^{(-)} | \Delta V_{pF} | \varphi_d \varphi_A \chi_{dA}^{(+)} \rangle = \langle I_A^F \chi_{pF}^{(-)} | \Delta V_{pF} | \varphi_d \chi_{dA}^{(+)} \rangle \quad (1)$$

$$M^{(prior)} = \langle \varphi_F \chi_{pF}^{(-)} | \Delta V_{dA} | \varphi_d \varphi_A \chi_{dA}^{(+)} \rangle = \langle I_A^F \chi_{pF}^{(-)} | \Delta V_{dA} | \varphi_d \chi_{dA}^{(+)} \rangle \quad (2)$$

where φ_A and φ_F denote the wave functions of the initial (A) and final ($F = A + n$) nuclei, respectively, and $I_A^F(r_{nA}) = \langle \varphi_A | \varphi_F \rangle$ is the associated overlap function, which depends on the coordinate of interest, r_{nA} . The distorted waves in the entrance and exit channels are given by χ_{dA} and χ_{pF} , respectively, and φ_d is the deuteron wave function. Post and prior forms require different transition operators

$$\Delta V_{pF} = V_{pn} + V_{pA} - U_{pF}, \text{ (post)} \quad (3)$$

$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}, \text{ (prior)} \quad (4)$$

respectively. In the post form, ΔV_{pF} contains the interaction between the proton and the target nucleus (V_{pA}), the proton-neutron interaction (V_{pn}), and the optical potential for the exit channel (U_{pF}), while in the prior form, ΔV_{dA} contains the interaction between the neutron and the target (V_{nA}), the proton-neutron interaction (V_{pn}), and the optical potential in the entrance channel (U_{dA}). In principle, post and prior formulations give numerically the same matrix element, but in practice small differences arise from ambiguities in the choice of the potentials used and the introduction of additional approximations.

In both formulations, the transition matrix element can be separated into two parts,

$$M^{(DWBA)} = M_{int}(0, R_{cut}) + M_{ext}(R_{cut}, \infty), \quad (5)$$

where R_{cut} refers to the specific value chosen for the coordinate r_{nA} , for which to carry out the separation. The interior term involves an integration from $r_{nA} = 0$ to $r_{nA} = R_{cut}$, while the exterior term involves an integration from $r_{nA} = R_{cut}$ to very large radii, $r_{nA} \rightarrow \infty$. Below, in Section 3, we will illustrate that for typical (d,p) transfer reactions and cutoff radii roughly equal to the size of the target nucleus, the main contributions to the post form matrix element come from the exterior region, while the prior form matrix element is dominated by interior contributions. This is significant, as in Ref. [1], it was further demonstrated that the post-form exterior matrix element can be expressed in terms of a surface term plus a prior-form exterior matrix element, so that the final expression for the matrix element is:

$$M^{(DWBA)} = M_{int}^{(post)}(0, R_{cut}) + M_{surf}(R_{cut}) + M_{ext}^{(prior)}(R_{cut}, \infty). \quad (6)$$

The same result is obtained if one starts with a consideration of the prior form and expresses the interior part in terms of a surface integral and remaining contributions from the interior. A dominant surface matrix element would provide several advantages: a) It would reduce the dependence of the cross section calculations on the model used for the interior portion of the overlap function; b) it would reduce the impact of the slow convergence of calculations of the exterior term when resonances are considered; and c) it would establish a useful link between resonance properties and transfer observables, since the surface term $M_{surf}(R_{cut})$, which is evaluated at a specific radius (R_{cut}), can be parameterized in terms of quantities that are familiar from traditional R-matrix approaches, namely a channel radius (here the cutoff radius R_{cut}), logarithmic boundary conditions (here logarithmic derivatives of known Hankel functions), and reduced-width amplitudes (here related to the asymptotic normalization of the overlap function). The contributions to typical cross sections arising from the three different terms in Eq. 6 will be discussed in Section 3 below.

3. Insights and implications

The separation into interior and exterior regions, and subsequent introduction of a surface term, can lead to improved descriptions of (d,p) reactions, in particular in cases where the final state is a resonance.

3.1. Interior and exterior contributions to the transition matrix element

Transfer experiments are often carried out under conditions that presumably make the reaction peripheral, *i.e.* they probe the overlap function in the region of the nuclear surface. The primary contributions to the transfer matrix elements should therefore come from r_{nA} values that are roughly equal to the size of the target nucleus. Figure 1, however, shows that the importance of a particular nuclear region depends on the form (post or prior) chosen. Shown are the contributions to the transfer cross section arising from the interior term in Eq. 5 only. Both post and prior forms are considered. The calculated peak cross section, normalized to the peak cross section obtained in the full calculation, is given as a function of the cutoff radius R_{cut} . We observe that the contributions for radii below 4 fm are small. In particular in the post formalism, the integration has to be carried out to fairly large radii in order to account for the majority of the cross section. This illustrates that the post form is more sensitive to the overlap function at larger radii (*i.e.* dominated by exterior contributions), while the prior formalism is more sensitive to the overlap at smaller radii (*i.e.* dominated by interior contributions). The reason for this can be found in the difference in the structure of the transition operators, Eqs. 3 and 4. Of course, as long as no further approximations are made, and both interior and exterior contributions are taken into account, both forms give the same result. Additional calculations, for (d,p) reactions on other nuclei (C, Ca, Pb), with various deuteron beam energies, give similar results. The findings also apply to reactions that populate resonances, which are found to be sensitive to larger values of r_{nA} , *i.e.* more peripheral regions of the nucleus. The interior contributions from the post-form matrix element were found to be small for all cases considered.

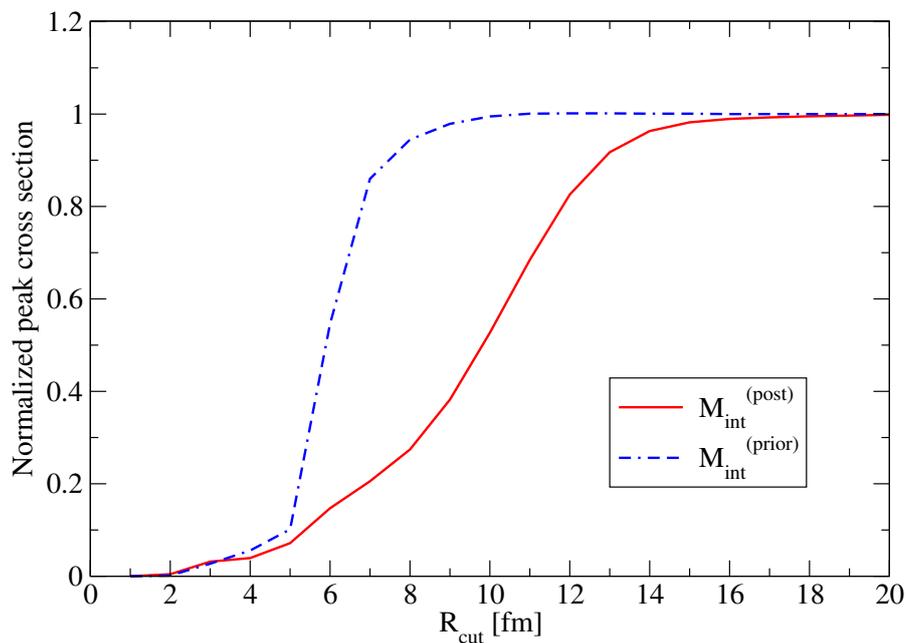


Figure 1. Examination of the role of interior contributions, for the $^{90}\text{Zr}(\text{d,p})$ transfer reaction to the $5/2^+$ ground state in ^{91}Zr . Shown is the peak cross section obtained by including only the interior term (M_{int}) in the transition matrix element, as a function of the cutoff radius R_{cut} . The cross sections are normalized relative to the peak cross sections obtained in the full calculation. Results are given for both the post and prior formalisms.

3.2. Role of the surface term

The smallness of the interior contribution to post-form transition matrix element supports the introduction of the surface term, as shown in Eq. 6. The replacement $M_{ext}^{(post)}(R_{cut}, \infty) \rightarrow M_{surf}(R_{cut}) + M_{ext}^{(prior)}(R_{cut}, \infty)$ is illustrated in Figure 2. Shown are the contributions to the cross section arising from the three terms in Eq. 6, as function of the surface radius R_{cut} . A region (6-9 fm) can be identified in which the contributions from the interior post and exterior prior terms are (relatively) small and the surface term dominates. Also shown, for reference purposes, is the exterior post contribution that was replaced (dotted light-colored curve). Similar results have been obtained for other target nuclei and bombarding energies. Reactions leading to ground states, low-lying excited states, as well as resonances have been considered, with similar findings. For resonance reactions, it was observed that the surface term tends to be peaked at larger radii, *i.e.* transfers to resonance states are more peripheral.

While the surface contribution shown in Fig. 2 is dominant around 7-8 fm, it can be observed that there are non-negligible contributions from both the interior (post) and exterior (prior) terms. It was shown in Ref. [1] that the exterior prior term does no longer appear when deuteron breakup effects are explicitly included via the continuum-discretized coupled-channels (CDCC) formalism [3], *i.e.* $M^{CDCC} = M_{int}^{CDCC}(0, R_{cut}) + M_{surf}^{CDCC}(R_{cut})$. The interior post contribution, on the other hand, requires further consideration. In the DWBA approach, this contribution can be minimized by appropriately optimizing the potentials in the transition operator, Eq. 3. This has to be done in a manner consistent both with the spirit of the DWBA and with the determination of the optical-model wave function. This matter is presently under investigation.

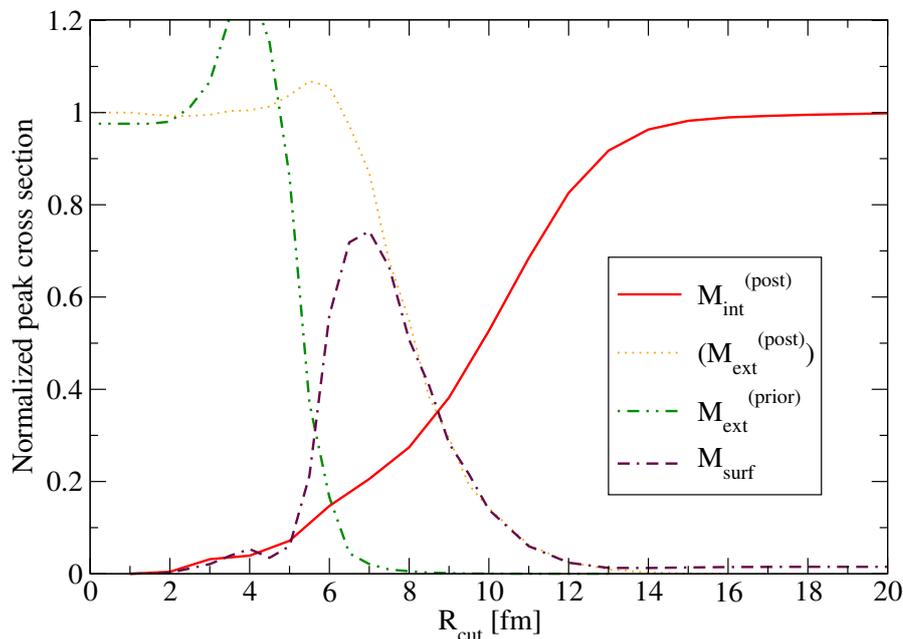


Figure 2. Contributions from the $M_{int}^{(post)}$, M_{surf} , $M_{ext}^{(prior)}$ terms to the $^{90}\text{Zr}(d,p)^{91}\text{Zr}_{gs}$ transfer cross section. Shown are the peak cross sections, normalized as in Fig. 1, as a function of the surface radius (R_{cut}). Also given is the exterior post contribution (dotted light-colored curve) that was replaced with the introduction of the surface term. The calculations were carried out using a modified version of the reaction code FRESKO [7].

4. Conclusions and outlook

One-nucleon transfer reactions, in particular (d,p) reactions, have played a central role in nuclear structure studies for many decades. Present theoretical descriptions of the underlying reaction mechanisms are insufficient for addressing the challenges and opportunities that are opening up with new radioactive beam facilities. We have discussed a new theoretical approach that was recently introduced specifically to address shortcomings in the description of transfers to resonance states. The method builds on ideas from the very successful R-matrix theory; in particular it uses a similar separation of the parameter space into interior and exterior regions, and introduces a parameterization that can be related to physical observables, which, in principle, makes it possible to extract meaningful spectroscopic information from experiments. We have illustrated some aspects of the approach, in particular the role of the surface term. The calculations were carried out in the DWBA approximation. Extensions to include deuteron break-up contributions in a CDCC version of the approach are underway.

Acknowledgments

This work, performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344, is supported by the DOE through the Topical Collaboration TORUS.

References

- [1] Mukhamedzhanov A M 2011 *Phys. Rev. C* **84**(4) 044616
- [2] Satchler G R 1983 *Direct Nuclear Reactions* (Oxford University Press)
- [3] Austern N, Iseri Y, Kamimura M, Kawai M, Rawitscher G and Yahiro M 1987 *Phys. Rep.* **154** 125–204
- [4] Thompson I J and Nunes F 2009 *Nuclear Reactions for Astrophysics: Principles, Calculation and Applications of Low-Energy Reactions* (Cambridge: Cambridge University Press)
- [5] Lane A M and Thomas R G 1958 *Rev. Mod. Phys.* **30** 257
- [6] Descouvemont P and Baye D 2010 *Reports on Progress in Physics* **73** 036301
- [7] Thompson I J 1988 *Computer Physics Reports* **7** 167–212